*Cone Penetration Testing 2022 – Gottardi & Tonni (eds)*


# Data-driven soil profile characterization using statistical methods and artificial intelligence algorithms

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ABSTRACT: CPT soil profile interpretation represents a fundamental aspect for subsoil stratigraphic reconstruction of complex geological contexts. In some situations, the soil profile may not exhibit evident boundary changes, making the interpretation more difficult. This crucial aspect plays a key role in the layers boundaries discontinuities identification and the construction of bi-dimensional and three-dimensional geotechnical models. In this paper, CPT and boreholes are used to calibrate and validate a massive and automated site characterization by combining statistical tools and artificial intelligence algorithms (AI). The procedure is applied in the complex stratigraphic context of Terre del Reno (Italy). The proposed data-driven analysis allows to combine the geological and geotechnical knowledge of the subsoil in an efficient and automatic way based on site-specific data, obtaining reliable and indispensable results for the construction of a robust and coherent geotechnical model of the subsoil.

## 1 INTRODUCTION

One of the subsoil stratigraphic reconstruction challenges is to identify homogeneous soil layers characterized by a different behaviour response. The detection of the strata boundaries represents a crucial aspect in the construction of two-dimensional and three-dimensional geotechnical models. The development of the data-driven analysis and the increasing availability of digitized geo-databases allow to improve the current methodologies to obtain an effective and extensive site characterization, providing a less subjective interpretation of the subsoil data (Phoon *et al*., 2021).

The main studies rely on statistic and probabilistic approaches based on the cone penetration test (CPT) data (Phoon *et al*. 2003, Facciorusso & Uzielli 2004, Uzielli 2008, Wang *et al*. 2013, Paolella *et al.* 2019, Shuku *et al*. 2020). The continuous measurements of soil parameters allow a statistical treatment for the identification of lithological discontinuities and the reconstruction of the stratigraphic profiles (Lo Presti *et al*. 2009). For this purpose, the CPT-based soil behaviour type index (Ic), and the correspondent soil behaviour type (SBT), are used to delineate soil boundaries (Robertson 2016).

This work developed a data-driven methodology for the subsoil stratigraphic reconstruction combining statistical tools and artificial intelligence algorithm (AI). The proposed procedure is applied to the complex stratigraphic context of Terre del Reno (Italy), extensively affected by seismic liquefaction phenomenon during the May 2012 earthquake sequence (Fioravante *et al.* 2013). From the geodatabase of the Emila Romagna Region, 102 pairs of CPT and boreholes were selected to calibrate and validate the method. The CPT profiles are analyzed, and the homogeneous layers are identified following statistical methods based on the spatial variability analysis of the measured parameters. Each statistically homogeneous layer is associated with the stratigraphic information extrapolated from closer boreholes, according to the Unified Soil Classification System (USCS). From the previous results, an artificial classifier is trained, and the automatic classification procedure of soil profile is tested on a complementary subset of data.

The proposed data-driven analysis combines the geological and geotechnical knowledge of the subsoil in an efficient and automatic way, based on site-specific data, to obtain reliable and indispensable results for the construction of a robust and coherent geotechnical model of the subsoil.

## 2 METHODOLOGY

The proposed approach aims to define a methodology for the subsoil stratigraphic recognition, applicable at the urban scale. In practice, this operation is based on the interpretation of a considerable amount of data

DOI: 10.1201/9781003308829-104

coming from different sources. The most diffused approach is to combine the borehole logs stratigraphy and Cone Penetration Test results in a deterministic way with a consequent subjectivity of the interpretation of available data.

The increasing availability of site-specific data collected, digitized and stored in the geodatabase allows to improve, even automatically, the consolidated methodologies for interpretation of subsoil data.

This procedure, summarized in Figure 1, provides a stratigraphic interpretation model applicable to all available surveys in the analyzed area, combining information from different investigation tests. In particular, the proposed methodology identifies the main lithologies from borehole log stratigraphies, the layer discontinuities from the sectioning statistical test of the CPT profiles and the automized stratigraphic classification of each statistically homogeneous layer from AI.

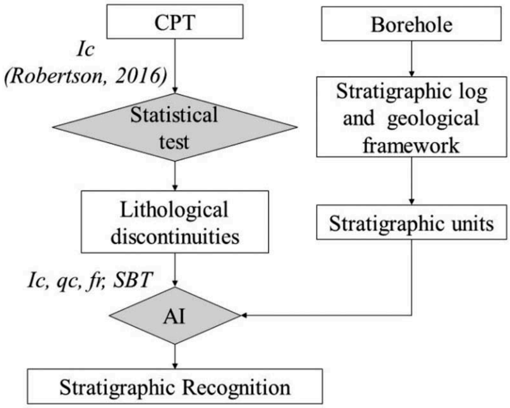



Figure 1. Proposed methodology for the subsoil stratigraphic recognition.

### 2.1 *Definition of stratigraphic units*

The definition of the main stratigraphic units detectable in the study area requires a detailed geological, hydrogeological, geomorphological and sedimentological framework.

The stratigraphic reference units for the selected case study are defined by combining the information from the geological framework and the borehole log stratigraphy. The detail of the stratigraphic model, its extension and stratigraphic classification are strictly related to the typology of the performed analysis (i.e. subsidence, liquefaction, design of structures and infrastructures). Therefore, the model (depth and thickness of the strata) and the stratigraphic units are defined according to the analyzed phenomenon (i.e., for a liquefaction risk assessment, the liquefiable layers).

### 2.2 *Soil boundaries discontinuities*

The homogeneous soil layers within the CPT profiles are identified based on the soil behaviour type index Ic (Robertson, 2016). The automatic procedure provides an accurate interpretation of the CPT tests considering the spatial correlation of the considered values along with the vertical profile.

This statistical test verifies the equality of the means and the variance of two subsets of data, according to the procedure shown in Figure 2. The two subsets of data (namely $\Omega_1$ and $\Omega_2$, with size respectively equal to $n_1$ and $n_2$, average $\overline{Q}_1$ and $\overline{Q}_2$, and variance $\sigma_1{}^2$ and $\sigma_2{}^2$) are identified along with the vertical CPT profile with a moving window $W_{d0}$ dived by $d_0$. The T ratio (Equation 1) and the intra-class correlation coefficient $\rho_I$ (Equation 2) are calculated along with the vertical CPT profile.

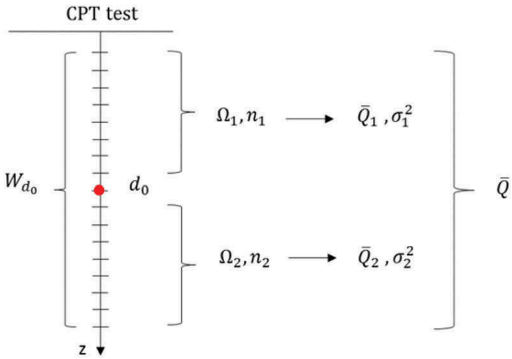



Figure 2. Definition of the two subsets of relevant parameters along the vertical axis of CPT test (from Spacagna et al, 2015).

$$T = \frac{\bar{Q}_1 - \bar{Q}_2}{\gamma_w} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \tag{1}$$

Where:

$$\gamma_w = \frac{n_1}{n_1 + n_2 - 1}\sigma_1^2 + \frac{n_2}{n_1 + n_2 - 1}\sigma_2^2 \tag{2}$$

$$\sigma_1^2 = \frac{1}{(n_1 - 1)} \sum_{i=1}^{n_1} \left(Q_i - \bar{Q}_1\right)^2 \tag{3}$$

$$\sigma_2^2 = \frac{1}{(n_2 - 1)} \sum_{i=1}^{n_2} \left(Q_i - \bar{Q}_2\right)^2 \tag{4}$$

$$\rho_I = \frac{\gamma_b^2}{\gamma_b^2 + \gamma_w^2} \tag{5}$$

Where:

$$\gamma_b^2 = \frac{1}{n_1 + n_2 - 1} \sum_{i=1}^{n_1+n_2} (Q_i - \bar{Q})^2 \qquad (6)$$

where $\overline{Q}$ is the average of the data $Q_i$ belonging to the window $w_{d0}$, with i=1,2, …, $(n_1+n_2)$.

To define the window $W_{d0}$, the geostatistical approach proposed by Spacagna *et al*. (2015) suggests to calculate the one-dimensional experimental variogram of the variable (Chilès & Delfinet 2012), with a lag equal to the minimum distance of measured point, following the Equation 7.

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (z(i) - z(i+h))^2 \qquad (7)$$

where z(i) is the value of the considered variable at a location, z(i+h) is the value of the variable at the distance h, and N(h) is the number of couples of points with a distance equal to h. This spatial correlation of the variable is modelled with a theoretical function. In the present study, the spherical model (Chilès & Delfinet 2012) is adopted to interpolate the spatial correlation (Equation 8).

$$\gamma(h) = \begin{cases} C\left(\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3}\right) & 0 \le h \le a \\ C & h > a \end{cases} \qquad (8)$$

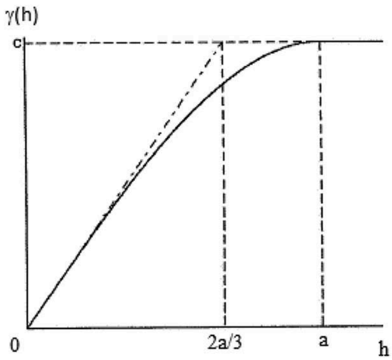



Figure 3. Spherical variogram.

In general, the variogram $\gamma(h)$ is an increasing function with the distance h. The model introduces two parameters. The *sill C* describes the level of spatial variability, and the *range a* represents the maximum distance at which spatial correlation is observed.

The amplitude of $w_{d0}$ used for the statistical test is defined as a ratio of the range a. The T ratio and $\rho_I$ are calculated for each point $d_0$, implementing two new vertical profiles. The higher values of T ratio and $\rho_I$ correspond to a change of behaviour of the CPT-based parameters Ic.

The critical value of the parameter T ratio ($t_c$) is evaluated considering the 90% confidence interval of the T ratio distribution and is calculated following Equation 9.

$$t_c = \mu_{T\,ratio} \pm 1,65\, \sigma_{T\,ratio} \qquad (9)$$

where $\mu_{Tratio}$ and $\sigma_{Tratio}$ are respectively mean and standard deviation of the distribution of the T ratio values along the vertical profile. The depth to which T ratio values fall outside the confidence interval represents a change of behaviour along the CPT profile.

The critical value of $\rho_{Ic}$ is calculated following the Equation (10) proposed by Herzagy, Mayne, and Rouhani (1996).

$$\rho_{Ic} = \mu_{\rho_I} + 1,65\, \sigma_{\rho_I} \qquad (10)$$

where $\mu_{\rho I}$ and $\sigma_{\rho I}$ are respectively the mean and standard deviation of the distribution of the $\rho_I$ values along the vertical profile. The depth to which $\rho_I$ values are higher than $\rho_{Ic}$ represents a change of behaviour along the CPT profile.

The transition between two different homogeneous layers is assumed at the $d_0$ depth points which both critical conditions occur simultaneously.

The described algorithm is implemented with open-source R software (R Core Team, 2021).

## 2.3 *Stratigraphic recognition with AI*

In the proposed methodology, the stratigraphic recognition, basically the attribution at each statistically homogeneous layers the correspondent stratigraphic unit, is automatically carried out with artificial intelligence (AI), studying the underlying relation between cone penetration tests information and stratigraphic properties. Reale *et al*. (2018) proposed an automatic classification of fine-grained soils using CPT measurements and Artificial Neural Network. Ching, Wu & Phoon (2020) and Ching *et al*. (2020) proposed a Hierarchical Bayesian Model (HBM) to construct transformation models for soil and rock properties.

The algorithms are trained comparing the outcomes derived from couples of CPT-boreholes considered spatially correlated, providing a complementary description of the subsoil. The CPT-borehole distance has been chosen based on the analysis of the spatial structure of cone tip resistance in the horizontal direction. A measure of the spatial correlation is the scale of fluctuation, representing the maximum distance over which the points are significantly related (Chilès & Delfinet, 2012).

The calibration procedure is structured as follows: first, the couples of complementary CPT and boreholes are extracted from the geodatabase; once CPT profiles are processed, to each statistically homogeneous layer is associated the stratigraphic unit reported in the complementary borehole; last, the CPT output parameters and their correspondent stratigraphic recognition are used to train, test and validate various algorithms.

The artificial classifier characterized by the highest efficiency is selected and applied to automatically assign the stratigraphic units to the remained sectioned cone penetration test profiles distributed over the studied area.

## 3 CASE STUDY

The Terre del Reno municipality is located in the southern Po river plain in the Emilia Romagna region (Italy). The area was struck by an intense seismic sequence associated with compression fault ruptures in 2012. The main shock of May 20$^{th}$ 2012 ($M_w$6.1) produces extensive liquefaction phenomena due to subsoil composition, geologic history and the shallow depth of the groundwater table (Fioravante et al. 2013). The municipality covers an area of 51 km$^2$, along a former branch of Reno River and is divided into three main districts: Sant'Agostino, San Carlo and Mirabello (Figure 4a).

The San Carlo area is characterized by a complex stratigraphic context due to the depositional history of the Reno river, highlighted by the significant number of liquefactions induced damages (Figure 4b). The calibration of the methodology has been carried out by analyzing the entire available geodatabase on the territory. However, in this work, the stratigraphic recognition is limited to the San Carlo area.

### 3.1 *Definition of stratigraphic units in the municipality of the Terre del Reno*

The subsoil of San Carlo is characterized by a relatively recent geologic history, an intensive depositional sequence of the Reno river and a shallow groundwater table (Figure 4c). The depositional activity of the river left superficial layers of loose sand with a thickness of a few meters. In the same period, the population constructed artificial levees mixing sand with silt to regulate the fluvial regime and limit flooding. The urban area is mainly built near those paleo-channel and paleo-levees.

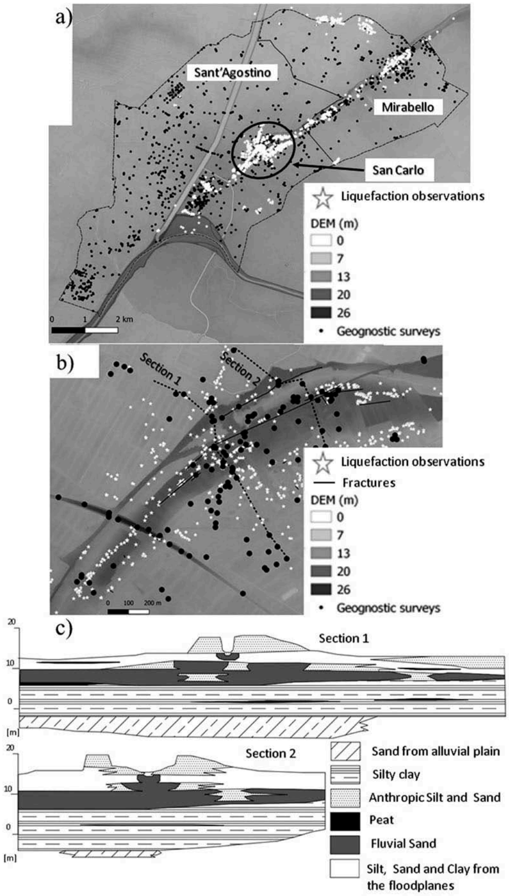



Figure 4. A) Terre del Reno municipality b) San Carlo district, c) Geological sections (adapted from Calabrese *et al.*, 2012).

The deepest layers are constituted of sediments deposited by Apennine and Alpine rivers, interspersed with marine accretion. These alternated processes are ruled by the regression and progression of the Adriatic Sea in the upper Pleistocene-Holocene (Romeo 2012).

The territory of Terre del Reno is covered by around 1700 geognostic surveys, highlighted in Figure 4a. The definition of the stratigraphic units has been carried out analyzing the description reported in 252 boreholes logs distributed all over the municipality. From the geological and sedimentological characterization, four stratigraphic units have been identified in the upper 20 m of the subsoil:

- Unit #1: silt, silty clay, clayey silt;
- Unit #2: sandy silt, silty sand;
- Unit #3: fine and medium clean sand;
- Unit #4: clay, organic clay, organic material.

### 3.2 *CPT profiles processing*

The soil boundaries discontinuities recognition has been applied to the district of San Carlo. In the selected area, there is a high density of surveys (Figure 4b). For 369 investigation tests distributed over an area of 3 $km^2$, the available boreholes and cone penetration tests reach 82.5%. In particular, the mechanical CPT correspond to 35.5% and the electrical ones to 32% of the total.

The implemented tool provides automatic sectioning of CPT profiles into statistically homogeneous layers.

An example of sectioned CPT profile is shown in Figure 5. From the CPT parameters, the Ic profile is computed. The statistical parameters $T_{ratio}$ and $\rho$ are calculated, and the critical thresholds are estimated. The horizontal dashed lines identify the transition between two different homogeneous layers.

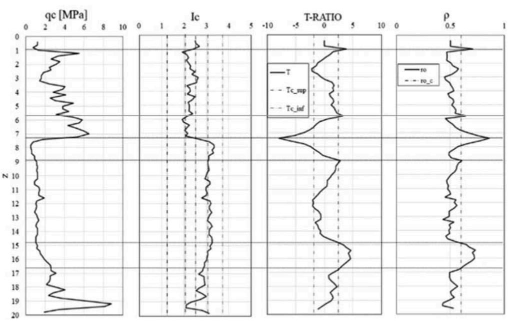



Figure 5. Example of sectioned CPT profile.

For each stratum, the following parameters are computed:

- the Soil Behaviour Type SBT,
- the mean and standard deviation of the Ic values,
- the mean and standard deviation of the friction ratio, FR, defined as ratio between sleeve friction and cone resistance.

### 3.3 *Training and application of the artificial intelligence algorithm*

At this stage, information reported in boreholes and sectioned CPT must be cross-correlated to perform an automatic stratigraphic recognition. The calibration of the artificial intelligence algorithm has been performed comparing the outcomes derived from couples of CPT-borehole considered spatially correlated. Due to the complex stratigraphic context in the investigated area of Terre del Reno (Fe), each CPT profile has been considered representative of a circular area having a radius equal to 30 m. This distance has been chosen because the cone tip resistance of the selected site is characterized by a correlation length equal to $\Theta$ = 16.5 m in the horizontal direction and a scale of fluctuation $\delta$ = 33 m, representing the maximum distance over which the points are significantly related.

All boreholes located in the defined distance are classified as complementary of the cone penetration tests. The total number of couples CPT-boreholes is equal to 132, but 30 have been deleted due to the poor quality and reliability of the subsoil description. The remaining 102 pairs constitute the starting point for the calibration of the algorithm. The sectioned CPT and the complementary boreholes have been compared.

Besides the output sectioning CPT parameters, at each statically homogeneous layer has been attributed the stratigraphic unit recognized in the complementary boreholes. These parameters are the input values for the calibration of a classifier capable to identify the corresponding stratigraphic unit.

The choice of the algorithm involves the estimation of the efficiency between 32 different artificial classifiers, implemented in the *Classification Learner APP* available in MATLAB R2021b, with a 10 folds cross-validation procedure (Stone 1974). This procedure is suggested to avoid overfitting and to estimate the accuracy obtained within the 10 iterations. Cross-validation divides the dataset into 10 folders of the same size: 9 folders are used to train the classifier while one is used to validate it. This procedure is iterated 10 times, training and testing each folder. The best classifier is a linear discriminant, characterized by an efficiency equal to 81.6%. Linear Discriminant Analysis (LDA) has been proposed by Fischer in 1936. It consists in finding the hyperplane projection that minimizes the interclass variance and maximizes the distance between the projected means of the classes.

In Figure 6a is shown the efficiency of the algorithm to classify the four units, attributing the percentage of success and misclassification. The positive prediction is expressed as Positive Predicted Values (PPV), and the negative classification is expressed as False Discovery Rates (FDR).

Figure 6b consist in a report showing a summary of prediction results of the mutual classification between classes, the grey cells correspond to the PPV and the sum of the remaining elements along the columns is equal to the FDR. For example, the first column summarizes the detail of the layers automatically classified as Unit #1: truly classified in 75.4% of occurrences; Unit #2 has been mistaken with Unit #1 in 15.8% of manifestations; Unit #1 has been attributed to the real Unit #3 in 1.2% and in the remaining 7.6%, Unit #4 has been classified as Unit #1. In particular, the mutual misclassification between sands and clays never occurs. Furthermore, the lower PPV values are associated with Units #1 (silt, silty clay, clayey silt) and #2 (sandy silt, silty sand), located in the upper 10 m of the subsoil and characterized by a complex geological history.

a)

| | Predicted Unit | | | |
|---|---|---|---|---|
| | **1** | **2** | **3** | **4** |
| **PPV** | 75.4% | 74.9% | 83.2% | 92.9% |
| **FDR** | 24.6% | 25.1% | 16.8% | 7.1% |

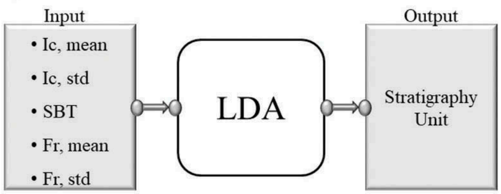
b)

| | | Predicted Unit | | | |
|---|---|---|---|---|---|
| | | **1** | **2** | **3** | **4** |
| True Unit | **1** | **75.4%** | 7.3% | 1.7% | 6.2% |
| | **2** | 15.8% | **74.9%** | 15.1% | 0.9% |
| | **3** | 1.2% | 17.4% | **83.2%** | 0.0% |
| | **4** | 7.6% | 0.4% | 0.0% | **92.9%** |

Legenda:
- 1: silt, silty clay, clayey silt;
- 2: sandy silt, silty sand;
- 3: fine and medium clean sand;
- 4: clay, organic clay, organic material.

Figure 6. Efficiency of the proposed LDA algorithm.

a)




b)

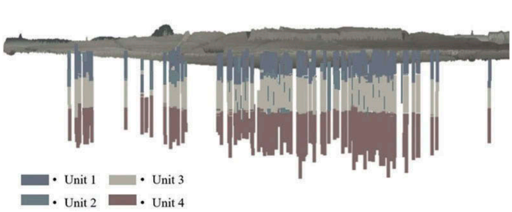



Figure 7. a) Input parameters and output for the selected LDA algorithm and b) application on the surveys disseminated all over the area.

Once the effectiveness was ascertained, the algorithm (Figure 7a) was applied to the remaining cone penetration tests, obtaining the stratigraphic recognition of the surveys disseminated on the investigated area (Figure 7b).

## 4 CONCLUSION

This work aims to propose a methodology for subsoil stratigraphic reconstruction based on the most diffused geotechnical survey, combining statistical tools and artificial intelligence algorithms. In particular, the proposed methodology identifies the layer discontinuities from the sectioning statistical test of the CPT profiles, the main lithologies from borehole log stratigraphies, and the automized stratigraphic classification of each statistically homogeneous layer from AI. The procedure has been implemented and validated in the complex geological context of the alluvial plain of Terre del Reno in Italy, characterized by a large geodatabase. The automatized methodology is easily replicable in other contexts for the stratigraphic recognition of the subsoil on a large scale, providing a valid tool for several applications as liquefaction risk analysis at an urban scale on structure and infrastructure (Paolella *et al.* 2021, Baris *et al.* 2021, Modoni et al. 2019) and subsidence study (Spacagna *et al.* 2020, Spacagna & Modoni, 2018). The data-driven analysis provides reliable and indispensable results for the construction of a robust and coherent geotechnical model of the subsoil. The next step concerns the interpolation of the stratigraphic surfaces to obtain a complete reconstruction of the subsoil.

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