A Novel Design Method of Four-Bar Linkages Mimicking the Human Knee Joint in the Sagittal Plane

Luciano Tomassi(✉), Chiara Lanni, and Giorgio Figliolini

DICEM, University of Cassino and Southern Lazio, Cassino, Italy
{luciano.tomassi,lanni,figliolini}@unicas.it

Abstract. A novel design method of four-bar linkages mimicking the human knee joint in the sagittal plane is proposed. This is based on a practical and efficient experimental approach to detect and analyze the planar motion of the tibia link with respect to the fixed femur, by which three significant positions are selected for design purposes. Thus, the kinematic synthesis of a four-bar linkage with anthropomorphic characteristics is developed by approximating the human knee joint to a pair of conjugate profiles. Finally, the centrodes of the designed four-bar linkage are obtained.

Keywords: Human knee joint · Four-bar linkage · Kinematic synthesis · Centrodes

1 Introduction

Over the years, countless devices that were able to replicate the movement of the knee have been developed. One of the first scientific works on knee kinematics was that of Freudenstein, in 1969 [1]. In it, we can find themes and methodologies such as the mathematical description of the two-dimensional kinematics of the knee and the use of centrodes, that have been pioneering in the field related to the development of mechanisms for the knee. Without a doubt, the most used mechanism to approximate the movement of the knee is the four-bar linkage. In 1994 Radcliffe [2] provided a solution of a four-bar linkage as a knee prosthesis. Dathe et al. [3] offer an accurate description of the movement of the human knee as a four-bar on the sagittal plane. Also, Sancisi et al. [4] and Bapat et al. [5] provide a methodology for optimizing four-bars for the knee starting from the acquisition of experimental data. Etoundi et al. [6], with the aim of creating an anthropomorphic mechanism for the human knee, proposed a prosthesis based on the use of an antiparallelogram linkage.

In this paper, a novel design method of four-bar linkages mimicking the human knee joint in the sagittal plane is proposed. Thus, a first phase of experimental data acquisition was carried out. Afterwards, three design positions were selected: the first relating to the bent leg, with the subject sitting on a chair prepared for the test and with the legs slightly suspended from the ground; the last relating to the leg in full extension and

© The Author(s), under exclusive license to Springer Nature Switzerland AG 2022
V. Niola et al. (Eds.): IFToMM Italy 2022, MMS 122, pp. 586–592, 2022.
https://doi.org/10.1007/978-3-031-10776-4_67

an intermediate position. These three positions, shown in Fig. 1a by the segments *MN*, coupled with an analysis of the profiles of the bones in contact, allowed to define the three design positions of the four-bar coupler link, which was graphically determined.

The centrodes of the resulting mechanism were traced in order to compare them with the ones present in the scientific literature and validate the synthesis procedure.

2 Experimental Setup and Motion Analysis

The goal of designing a mechanism that is able to replicate as accurately as possible the movement of the human knee joint involves, in the first instance, a careful analysis of the real movement of the joint itself. The data acquisition methodology considered that the movement of the knee can be approximated to a two-dimensional motion. This approximation is more than reasonable because for the human anatomy the knee moves in a reference plane, the sagittal plane. The second key factor is that the synthesis of the mechanism is performed starting not from the continuous motion of the knee, but considering a certain number of finite positions acquired during its flexion-extension phase.

The advantage of referring to rigid positions acquired by the knee rather than directly to its continuous motion consists in being able to use very accurate kinematic synthesis techniques, which ensure the mathematical passage of the mechanism to be realized for the desired positions. Starting from this, the idea was to build the entire phase of kinematic synthesis on the image analysis, that represent different rigid positions assumed by the knee during the flexion-extension movement. Starting from position 1, relative to the fixed leg, the acquired movement is the natural one of the tibia, due to the action of the muscles and without the action of external forces. In the scientific literature, several approaches to acquire data related to knee movement can be found, as in [7, 8].

In this paper, suitable reference points for the development of the kinematic synthesis have been foreseen by using a specific device with a *T*-shape. For this purpose, a 3D printed object, that for brevity will be called *T,* was specially created (Fig. 1b). To make the *T* in solidarity with the tibia during the movement, it was mounted on a shin guard, which was connected to the tibia by means of Velcro bands. At this stage it was necessary to pay attention to the fact that the shin guard had to be located on the straight part of the tibia, as well as to the fact that the plane identified by the frontal face of the T was as parallel as possible to the sagittal plane. The part of the leg considered fixed during the motion is the femur which, always through Velcro bands, has been rigidly fixed to a chair specially prepared for the test.

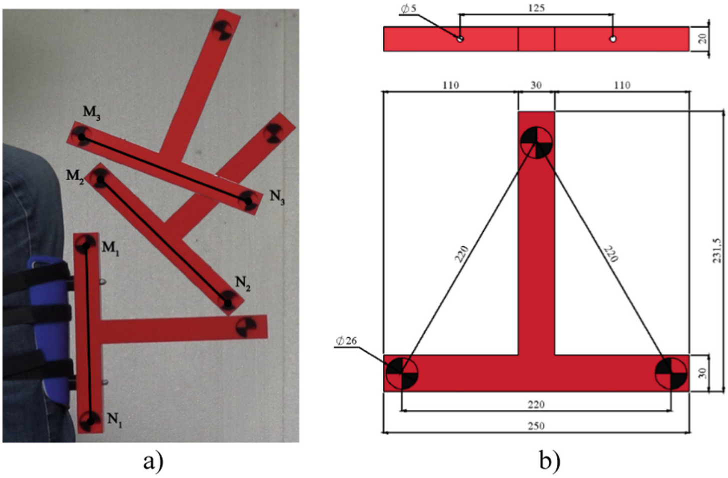


Fig. 1. Experimental setup: a) Three selected positions; b) Sizes of the experimental device.

In order to be able to choose the positions of the circle points, that are the revolute joints of the coupler link, that are in line with the anthropomorphic design specifications, it was necessary to study more in detail the movement of the hu-man knee. The anthropomorphism specification relates to the size of the four-bar and the position of the revolute joints, that must be located in the range of the human knee joint. Since the macroscopic flexion-extension movement of the knee is generated by the relative motion of the surfaces of the femur and the tibia in contact, by schematizing motion as a two-dimensional motion, it is possible to distinguish two conjugate profiles. The profiles were studied starting from an image related to a lateral X-ray of the human knee, taken from an atlas of the X-rays of the human body [9]. For this purpose, the image shown in Fig. 2a was processed. Thus, to find the positions of the circle points starting from the configuration of the bone profiles in contact, they have been approximated with two ellipsoids: the ellipsoid named with σ represents the knee profile, while the one named with s represents the tibia profile. Denoting with Ω_σ the center of curvature of the femur profile and with Ω_s the center of curvature of the tibia profile, a rigid link is introduced and is chosen to be the first position of the four-bar coupler link. The center of curvature of the two profiles coincides with the center of the two circle points.

This methodology, although approximate, made it possible to find the position of the coupler of the four-bar in the first configuration. The subsequent positions 2 and 3 of the coupler were identified by the *transport method*, starting from the positions taken by the T, being both belonging to the same moving plane (Fig. 2b). From this point on we proceeded with the synthesis of the four-bar linkage.

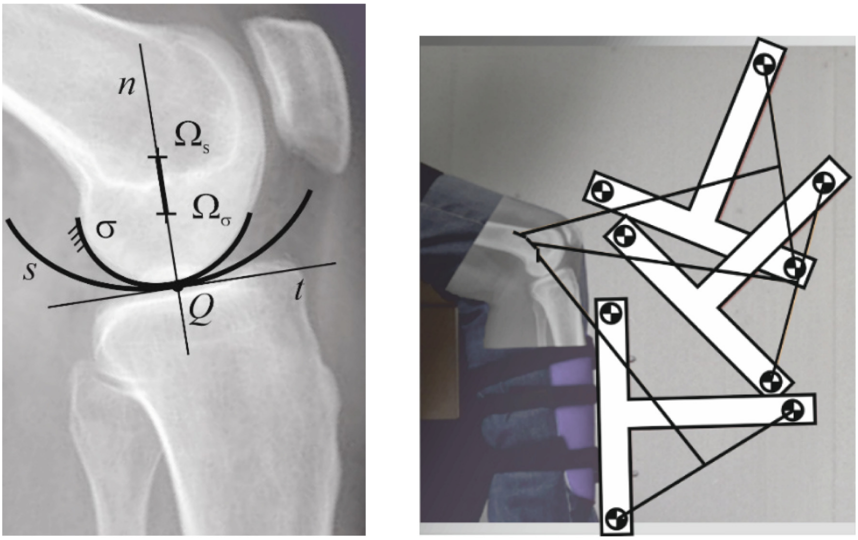


Fig. 2. Human knee joint: a) Conjugate profiles of femur-tibia pair; b) Transport method.

3 Kinematic Synthesis and Analysis

Once the circle point positions of the coupler of the four-bar were known, we proceeded with the determination, by graphic method, of the position of its center points. The graphic synthesis, shown in Fig. 4a, can be summarized as follows: fixed the circle points in positions 1, 2 and 3, the displacement vectors of point A and point B, relating to the displacements 1–2 and 2–3 of the points A and B, have been traced. The two medians relating to the displacement vectors of point A identify the finite center of rotation of point A, those relating to the displacement vectors of point B identify the finite center of rotation of point B. Putting the two fixed revolute joints in A_0 and B_0 in the aforementioned centers, the four-bar linkage is automatically defined. This graphic construction has been reported many years before in many textbooks of advanced Mechanics of Machines [10]. The obtained four-bar, whose kinematic model is shown in Fig. 3b, allows the passage through the three design positions in an exact way and replicates quite faithfully the overall movement of the knee.

In order to validate the obtained results, the four-bar centrodes were computed with a routine developed in MATLAB, based on the analytical method of the loop-closure equations [11, 12].

The position analysis is developed by the following loop-closure equation, Fig. 3b),

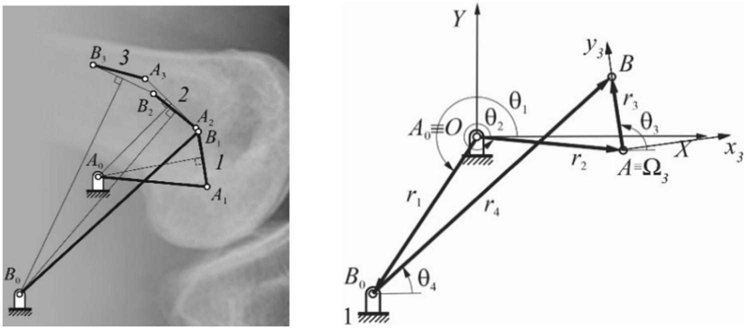
$$\mathbf{r}_1 + \mathbf{r}_4 = \mathbf{r}_2 + \mathbf{r}_3 \tag{1}$$

Thus, each vector $\mathbf{r}_i$ for $i = 1, \ldots, 4$ can be expressed in matrix form with respect to the fixed frame OXY as

$$\mathbf{r}_i = r_i\left[\cos\theta_{\mathrm{i}},\ \sin\theta_{\mathrm{i}}\right]^T \tag{2}$$

where r_i and θ_i are the magnitude and the counterclockwise oriented angles of $\mathbf{r}_i$ respectively, while T indicates the transpose. Angles θ_4 and θ_3 are obtained as function of the driving crank angle θ_2 by solving Eq. (2) as

$$\theta_4 = \tan^{-1}\frac{-B + \sigma\sqrt{B^2 - C^2 + A^2}}{C - A} \tag{3}$$



a) b)

Fig. 3. Four-bar linkage: a) graphical synthesis; b) kinematic sketch.

$$\theta_3 = \tan^{-1}\frac{r_1\sin\theta_1 + r_4\sin\theta_4 - r_2\sin\theta_2}{r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2} \tag{4}$$

where σ is equal to ±1 according to a suitable assembly mode and the coefficients A, B and C are obtained as function of the driving crank angle θ_2 by

$$A = 2\,r_1\,r_4\cos\theta_1 - 2\,r_2\,r_4\cos\theta_2 \qquad B = 2\,r_1\,r_4\,sin\theta_1 - 2\,r_2\,r_4\sin\theta_2 \tag{5}$$

$$C = r_1^2 + r_2^2 + r_4^2 - r_3^2 - 2r_1\,r_2(\cos\theta_1\cos\theta_2 + sin\theta_1\sin\theta_2) \tag{6}$$

Referring to Fig. 4, the fixed centrode λ_3 is the path traced by the instantaneous center of rotation P_3 of the coupler link 3 with respect the fixed frame OXY, while the moving centrode l_3 is the path traced by P_3 with respect the moving frame $\Omega_3x_3y_3$ that is attached to AB [13–15]. Thus, taking into account that the position of P_3 is obtained in agreement with both Chasles and Aronhold-Kennedy theorems, the fixed centrode λ_3 of the coupler link 3 can be expressed in vector form as follows

$$\mathbf{r}_{P3} = \frac{r_1(\sin\theta_1 - \cos\theta_1 tg\theta_4)}{tg\theta_2 - tg\theta_4}\begin{bmatrix}1 & tg\theta_2\end{bmatrix}^T \tag{7}$$

and the equation of the moving centrode l_3 is given by

$$\mathbf{r}^*_{P3} = r_3\frac{\sin(\theta_3 - \theta_4)}{\sin(\theta_4 - \theta_2)}\begin{bmatrix}\cos(\theta_2 - \theta_3) & \sin(\theta_2 - \theta_3)\end{bmatrix}^T \tag{8}$$

Figure 4 represent the centrodes in the passage for the three project positions 1, 2 and 3. Despite some differences, they appear quite similar in the shape of the centrodes presented in the scientific literature [1, 7].

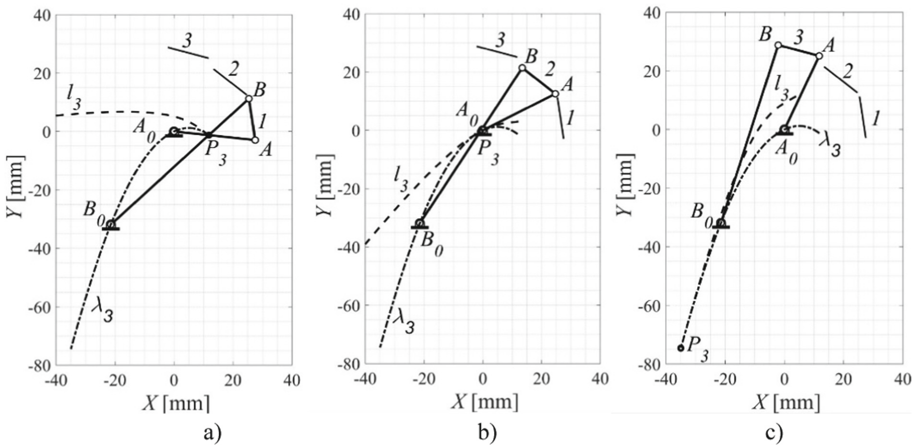


Fig. 4. Centrodes: a) starting position; b) intermediate position; c) extended position.

4 Conclusions

A novel methodology of kinematic synthesis of four-bar linkages mimicking the human knee joint has been proposed. After having produced the design specifications by means of experimental images acquisition, together with a study of the conjugated profiles of the bones of femur and tibia in contact, the four-bar linkage by graphic construction was obtained. The centrodes of the resulting mechanism were traced in order to compare them with the ones present in the scientific literature and validate the synthesis procedure.

References

1. Freudenstein, F.: Kinematics of the human knee joint. Bull. Math. Biophys. **31**, 215–232 (1969)
2. Radcliffe, C.W.: Four-bar linkage prosthetic knee mechanisms: kinematics, alignment and prescription criteria. Prosthet. Orthot. Int. **18**, 159–173 (1994)
3. Dathe, H., et al.: The description of the human knee as four-bar linkage. Acta Bioeng. Biomech. **18**(4), 107–115 (2016)
4. Sancisi, N., Caminati, R., Parenti-Castelli, V.: Optimal four-bar linkage for the stability and the motion of the human knee prostheses. In: Atti del XIX CONGRESSO dell'Associazione Italiana di Meccanica Teorica e Applicata, Ancona, pp. 1–10 (2009)
5. Bapat, G.M., Sujatha, S.: A method for optimal synthesis of a biomimetic four-bar linkage knee joint for KAFO. J. Biomim. Biomater. Biomed. Eng. **32**, 20–28 (2017)
6. Etoundi, A.C., et al.: A bio-inspired condylar knee joint for leg amputees and for knee implants. Des. Nat. **160**(6), 23–34 (2012)

7. Tsai, Y., et al.: On the centrodes of human knee joints using photographic method. Life Sci. J. **9**(1), 464–468 (2012)
8. Bernal-Torres, M.H.: Design and control of a new biomimetic transfemoral knee prosthesis using an echo-control scheme. J. Healthc. Eng. **2018** (2018)
9. Singh, H., Sheik, S.: Atlas on X-Ray and Angiographic Anatomy. Jaypee Brothers Medical Publishers (2013)
10. Suh, C.H., Radcliffe, C.W.: Kinematics and Mechanisms Design. Wiley, Hoboken (1978)
11. Figliolini, G., Angeles, J.: The spherical equivalent of Bresse's circles: the case of crossed double-crank linkages. ASME J. Mech. Robot. **9**(1), 011014 (2017)
12. Figliolini, G., Lanni, C., Kaur, R.: Kinematic synthesis of spherical four-bar linkages for five-poses rigid body guidance. In: Uhl, T. (eds.) IFToMM WC 2019. Mechanisms and Machine Science, vol. 73, pp. 639–648. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-20131-9_64
13. Figliolini, G., Lanni, C.: Geometric Loci for the kinematic analysis of planar mechanisms via the instantaneous geometric invariants. In: Gasparetto, A., Ceccarelli, M. (eds.) MEDER 2018. MMS, vol. 66, pp. 184–192. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-00365-4_22
14. Figliolini, G., Lanni, C., Tomassi, L.: Kinematic analysis of slider – crank mechanisms via the Bresse and Jerk's circles. In: Carcaterra, A., Paolone, A., Graziani, G. (eds.) AIMETA 2019. LNME, pp. 278–284. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-41057-5_23
15. Figliolini, G., Lanni, C., Angeles, J.: Kinematic analysis of the planar motion of vehicles when traveling along tractrix curves. ASME. J. Mech. Robot. **12**(5), 054502 (2020)