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Fractal analysis of market (in)efficiency during the COVID-19

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ABSTRACT

Using the multifractional Brownian motion as a model of the price dynamics, we analyze the impact of the COVID-19 pandemic on the efficiency of fifteen financial markets from Europe, US and Asia. We find that Asian markets (Hang Seng, Nikkei 225, Kospi) have recovered full efficiency, while European and US markets - after an initial rebound - have not yet returned to the pre-crisis level of efficiency. The inefficiency that currently characterizes US and European markets originates moderately high levels of volatility.

1. Introduction and motivation

As other recent and relevant financial shocks, also the ongoing global pandemic crisis is challenging the capability of traditional models to grasp and represent the complex mechanisms which rule the real financial markets. Precisely this limit constitutes perhaps the major weakness of the current mathematical models; based mostly on the assumption that past is fully discounted by current prices, they are unable to provide markets regulators with the directions to manage and stabilize financial markets when these experience large and sudden swings in quotes. Such events are reduced to “anomalies”, “outliers” or “singularities” and, as such, they are marginalized from the models built to describe the ubiquitous equilibrium of markets. Despite this mindset, financial crises do occur, with a frequency and a magnitude much larger than those predicted by standard theories. The COVID-19 pandemy is the current one; it has shaken financial markets, by prompting bursts of volatility and collapses in prices and stock indexes all over the world. The reconstruction of the context provided in Malkiel and Shiller (2020) well describes the point:

> “The gyrations of the world’s stock markets around the time of the onset of the coronavirus, Covid-19, pandemic have seemed illogical to many observers. The S&P 500 stock price index set an all-time record high on February 19, 2020, after over a month’s news of the epidemic. The World Health Organization had already labeled it a “public health emergency of international concern” on January 30 when the epidemic had already spread from China to 18 countries. This peak was a few days after news that the epidemic in China had already attacked 66,000 victims and caused 1,500 deaths. Wouldn’t you think the “smart money” and people on their toes would have seen trouble coming by then? Were they even paying attention? From that high, the S&P 500 fell 34% to a low on March 23, 2020. Maybe that drop could be justified from the dire pandemic narratives that were circulating then. But from there, with stay-at-home orders proliferating around the world, the S&P 500 rose 30% to April 30, 2020, amidst nightmarish figures, over three million cases worldwide and over 200,000 deaths. Wouldn’t you think that

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there wasn't much good news that would justify such a massive rebound? How can we explain what drove the market so rapidly up?"

The spikes of volatility along with large downward price movements are clear symptoms that market structure is suddenly changing, in the sense that the distribution of the trading strategies among the different time horizons is no longer capable to balance the excess of short positions opened by traders on a single or multiple investment horizons. This causes prices to fall and volatility to burst, Bianchi et al. (2020). At least partly, this mechanism is triggered by irrational behaviours that during the crises tend to prevail on the purely rational decision-making process, with an emphasis which is the more strong the more shocking the news. In these circumstances, markets deviate from equilibrium, meaning that their behaviour cannot be described by classical semimartingale-like models, because the assumptions supporting the informational efficiency, Fama (1970), are temporarily violated. Standard theories claim that such inefficiencies are istantaneously arbitraged away by financial markets, and efficiency is restored as soon as the new information is incorporated in the prices.

Given the mechanism above described, a relevant question arises: can the volatility level be related to the (in)stability of the market? In other words, can volatility be expressed in terms of departure from the value which can be considered "typical" of an efficient market? It is clear that a positive answer to this question could help in evaluating how long a financial shock can potentially last, and hence it could provide useful directions to both market operators and regulators.

To date, an impressive literature has focused on volatility and volatility clustering, and - since the pioneering work by Mandelbrot (1963) - many models have been proposed to account for the time-varying volatility of price variations: the α-stable distributions (Rachev and Mittnik (1993); Samorodnitsky and Taqqu (1994); Rachev and Mittnik (2000), Reussa et al. (2016)), whose main drawback is to be infinite variance models; the ARCH-GARCH-based family (Engle (1982); Bollerslev (1986); see Alexander (2001) or Francq and Zakoian (2019) for a complete review), whose residuals are anyway still fat-tailed and dependent, which is a clear symptom that volatility is not completely captured by the model; and the stochastic volatility models (Hull and White (1990), Heston (1993), Dupire (1994)), whose main idea is that volatility follows a diffusion process. Albeit the stochastic volatility models can be adapted to structural breaks involving for example both random times and random magnitudes, jumps and fat-tailed shocks or other forms of non-linearities, such a models remain difficult to estimate and forecast.

Regardless of the different approaches followed by the families of models quoted above, their common thread is that none of them makes an explicit link between volatility and market efficiency. So far, all the models have regarded volatility as to a relative measure; this means that only its variations are used to monitor the level of stress (i.e. of departure from efficiency) that the information flow injects the markets. Thus, volatility is evaluated high or low only with respect to its previous value, but a clear correspondence between market efficiency and an absolute measure of volatility is missing.

Motivated by this intrinsic limit of the current volatility models and by the requirement, pointed out by Engle and Patton (2001), that an appropriated volatility model should account for both mean-reversion and persistence in volatility dynamics (see, e.g., Fouque et al. (2000), Chen and Daigler (2006), Fouque and Lorig (2011), Patton and Sheppard (2015)), in this work we:

a. estimate the pointwise Hölder regularity $H(t)$ of fifteen stock indexes series before and after two large financial shocks (the COVID-19 and the benchmark represented by the 2007-2009 global financial crisis), assuming their dynamics to follow a multifractional Brownian motion. The advantage of this approach is twofold: on the one hand $H(t)$ is an equivalent measure of volatility homogeneous with respect to the financial markets; on the other hand, it directly expresses the intensity and the direction of the temporarily inefficiency of the series as compared to the efficient benchmark represented by the semimartingale case;
b. analyze the diverse reactions of the markets to the shock in terms of departure from the efficiency as well as of return to the pre-crisis level of efficiency;
c. analyze the cross-correlations of the series $H(t)$, in two time spans: one and two months before and after both the crises.

The paper is organized as follows: in Section 2 we recall the notion of pointwise regularity, the definition and the main properties of multifractional Brownian motion and the method that will be used in the application to estimate the pointwise regularity exponents of the stock indexes series; in Section 3 we develop the application; Section 4 concludes.

2. Model and estimation methodology

In order to achieve the goals a. - c. summarized in the Introduction, we need to shortly describe first some tools which will be used in the following. They are: the notion of pointwise regularity of a stochastic process (Section 2.1); the main properties of the multifractional Brownian motion, which is the model that we will assume for the dynamics of stock prices (Section 2.2); and, finally, how to estimate the pointwise regularity of a price sequence (Section 2.3).

2.1. Pointwise regularity

Given the stochastic process $X(t,\omega)$ with a.s. continuous and not differentiable trajectories over the real line $\mathbb{R}$, the local Hölder regularity of the trajectory $t \longmapsto X(t,\omega)$ with respect to some fixed point t can be measured through the *pointwise* Hölder exponent, defined as

$$\alpha_X(t,\omega) = \sup\left\{\alpha \geq 0 : \limsup_{h\to 0} \frac{|X(t+h,\omega) - X(t,\omega)|}{|h|^{\alpha}} = 0\right\}. \quad (1)$$

Relation (1) lends itself to the following geometrical interpretation: function X has exponent α at t_0 if, for any $\epsilon > 0$, there exists a neighborhood of t_0, $I(t_0)$, such that, for $t \in I(t_0)$, the graph of X is included in the envelope defined by $t \longmapsto X(t_0) - c|t - t_0|^{\alpha-\epsilon}$ and $t \longmapsto X(t_0) + c|t - t_0|^{\alpha+\epsilon}$ (see Lévy Véhel and Barriére (2008)).

If $X(t,\omega)$ is a Gaussian process, by virtue of zero-one law, there exists a non random quantity $\alpha_X(t)$ such that $\mathbb{P}(\alpha_X(t) = \alpha_X(t,\omega)) = 1$ (Ayache (2013)). In addition, when $X(t,\omega)$ is a Brownian motion, $\alpha_X = \frac{1}{2}$; values different from $\frac{1}{2}$ describe non-Markovian processes, whose smoothness is too high, when $\alpha_X \in \left(\frac{1}{2}, 1\right)$, or too low, when $\alpha_X \in \left(0, \frac{1}{2}\right)$, to satisfy the martingale property. This holds for Brownian as well as non Brownian well-behaved martingales. For these, the result follows from observing that if Z_t is a martingale difference with respect to $\mathcal{F}_t$ such that (a) $n^{-1}\sum_{t=1}^{n}\mathbb{E}(Z_t^2|\mathcal{F}_{t-1}) \xrightarrow{P} \nu$ for a positive constant ν, and (b) $n^{-1}\sum_{t=1}^{n}\mathbb{E}(Z_t^2\chi_{|Z_t|>\epsilon\sqrt{n}}\Big|\mathcal{F}_{t-1}) \xrightarrow{P} 0$ for every $\epsilon > 0$, then $\sqrt{n}\overline{Z}_n \xrightarrow{d} \mathcal{N}(0,\nu)$ (here, χ denotes the indicator function and $\overline{Z}_n$ the average of Z). Given the convergence to the normal law, the proof which covers the Brownian case applies to non Brownian well-behaved martingales, see e.g. Revuz and Yor (1999). In particular, the quadratic variation of the process can be proven to be zero, if $\alpha_X > \frac{1}{2}$ and infinite, if $\alpha_X < \frac{1}{2}$.

2.2. *Multifractional Brownian motion*

It is well-lnown that fractional Brownian motion (fBm or $B_H(t)$) (Kolmogorov (1940), Mandelbrot and Van Ness (1968)) is maybe one major example of stochastic process whose pointwise Hölder exponent is constant along its paths and controlled by the parameter $H \in (0,1)$. Indeed, at each point, $\alpha_{B_H}(t,\omega) = H$ almost surely (when $H = \frac{1}{2}$, the fBm reduces to the Brownian motion). The fact that H is constant along the paths of the fBm constitutes a major drawback in modelling those real world phenomena whose pointwise regularity changes, even abruptly, from point to point (as it happens, for example, for financial time series). This led to generalize the fBm into the multifractional Brownian motion (mBm), a Gaussian process whose pointwise Hölder exponent can be tuned at each point (Péltier and Lévy Véhel (1995); Benassi et al. (1997)). The extension is obtained by replacing the constant parameter H by a regularity function $H(t)$, subject to some technical conditions[1]. Denoted by $\mathbb{W}$ the real Brownian measure, a non anticipative moving average representation of the mBm reads as

$$B_{H(t)}(t) = \int_{-\infty}^{0}\left[(t-u)^{H(t)-\frac{1}{2}} - (-u)^{H(t)-\frac{1}{2}}\right]\mathbb{W}(du) + \int_{0}^{t}\left[(t-u)^{H(t)-\frac{1}{2}}\right]\mathbb{W}(du) \tag{2}$$

Likewise the fBm, also the pointwise regularity at t of the mBm equals the value of its functional parameter at t, namely $\alpha_{B_{H(t)}}(t,\omega) = H(t)$ almost surely (Ayache (2013)). This means that $H(t)$ is strictly related to the standard deviation at the same time t. Figure 1 makes this relation explicit: an exponential law of the form $\sigma(t) = ae^{bH(t)}$ can be established between the standard deviation (historical volatility) σ and the functional parameter of the mBm, both referred to time t.

Thus, under the assumption that financial markets to follow a dynamics that can be modelled by the mBm, our work exploits this relation to characterize the behaviour of a financial time series. For this purpose, how to estimate $H(t)$ from a time series of prices (that are assumed to behave as an mBm) becomes a central issue. To this aim, it comes in handy the following property of the mBm, stating that at each time t_0 the process is locally asymptotically self-similar[2] with parameter $H(t_0)$. In fact, setting $Y(t_0, \epsilon u) = B_{H(t_0+\epsilon u)}(t_0 + \epsilon u) - B_{H(t_0)}(t_0)$, it can be proved that

$$\lim_{\epsilon \to 0^+} \epsilon^{-H(t_0)} Y(t_0, \epsilon u) \stackrel{d}{=} B_{H(t_0)}(u) \tag{3}$$

where $\stackrel{d}{=}$ denotes equality in distribution and $u \in \mathbb{R}$. Relation (3) states that at any point t there exists an fBm with parameter $H(t_0)$ tangent to the mBm or, equivalently, in a neighborhood of t the mBm behaves like an fBm of parameter $H(t_0)$. Since $B_{H(t_0)}(u) \sim \mathcal{N}(0, u^{2H(t_0)})$, the (infinitesimal) increment of the mBm at time t_0 - normalized by $\epsilon^{H(t_0)}$ - distributes with mean 0 and variance $u^{2H(t_0)}$ $(u \in \mathbb{R}, \epsilon \to 0^+)$. This property can be exploited to build an estimator of $H(t)$ and, hence, of $\alpha_{B_{H(t)}}(t)$.

2.3. *Estimation of the pointwise regularity of mBm.*

Based on adaptations of the asymptotic estimators available for the fBm, several methods have been proposed in literature to estimate $H(t)$ (see, e.g., Lux and Segnon (2018), Garcin (2017)). They generally involve variation statistics (in particular, second order variations). A detailed description of these estimators is beyond the scope of this work and can be found in several references (see, e.g. Istas and Lang (1997), Kent and Wood (1997), Benassi et al. (2000), Coeurjolly (2001), Coeurjolly (2005), Bianchi (2005), Bianchi

[1] $H : (0,\infty) \to (0,1)$ is a Hölderian function of order β, i.e. such that on each compact interval $T \subset \mathbb{R}$, for all $t, s \in T$, $|H(t) - H(s)| \le c|t - s|^{\beta}$, with $c > 0$ and $\beta > \max_{t\in T} H(t)$.

[2] We recall that the stochastic process $\{X(t), t \in T\}$ is said *self-similar* with parameter H if for any $a > 0$ $\{X(at) \stackrel{d}{=} a^H X(t)\}$, where the equality holds for the finite-dimensional distributions of the process (see, e.g. Embrechts and Maejima (2002)).

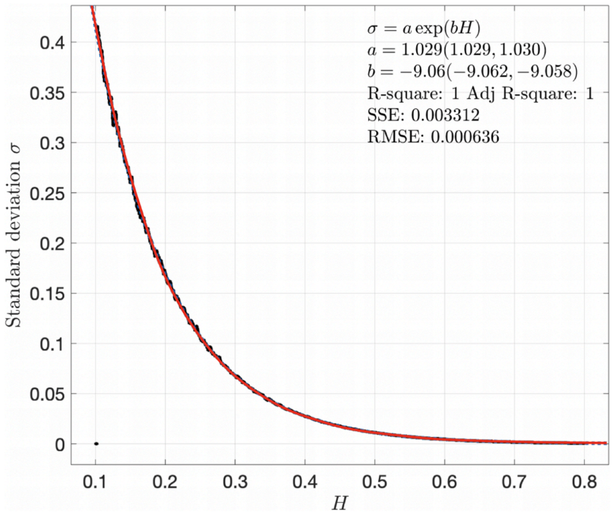


Fig. 1. Relation between the estimated standard deviation and the Hölder exponent. The standard deviation $\sigma(t)$ was estimated on a rolling window of 30 data along the path of an mBm of assigned functional parameter $H(t)$. The values of $H(t)$ were averaged in the same window to make the comparison possible. The Figure displays the average of 100 simulations of length 8,192 (dots) and the fitting curve (red line). The two plots are virtually undistinguishable since it is an almost perfect match, as revealed by the fitting parameters in the legend (the values in parentheses are the coefficients of the fitting curve at 95% bounds).

et al. (2013), Pianese et al. (2018)) Frezza (2018). For this reason, in the following, we will not go into the details of the estimation procedure, but we will merely use the estimator proposed by Pianese et al. (2018). Merging the unbiased, large-variance estimator introduced by Istas and Lang (1997) and Benassi et al. (1998) with the biased, low-variance estimator deduced in Bianchi et al. (2013), they obtain the unbiased, low-variance moving window estimator $\widehat{H}_{\nu,q,n}(t,a)$; ν indicates the size of the moving window, q is the differencing lag, n is the size of the sample, and a is a filter which acts to make the sequence locally stationary and to weaken the dependence between data.

$\widehat{H}_{\nu,q,n}(t,a)$ is unbiased and normally distributed, and its variance is known in closed form when $H(t)=\frac{1}{2}$, that is when the martingale condition holds. The confidence bounds with respect to $\frac{1}{2}$ can therefore be calculated as

$$\Phi(z) := \Phi_{\widehat{H}_{\nu,q,n}(t,a)|H(t)=\frac{1}{2}}(z) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z} e^{-\frac{(x-1/2)^2}{2\sigma^2}}dx \tag{4}$$

where $\sigma = \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{2k+1}{2}\right)-\Gamma^2\left(\frac{k+1}{2}\right)}{\nu k^2\log^2(n-1)\Gamma^2\left(\frac{k+1}{2}\right)}\right)^{1/2}$. Thus, at a given p-level level α, $H(t)$ does not significantly deviates from $\frac{1}{2}$ if it belongs to the interval $E^{\alpha} := [\Phi^{-1}(\alpha/2), \Phi^{-1}(1-\alpha/2)]$. Likewise, if X is the log-price process, efficiency cannot be rejected at $(1-\alpha)\%$ provided that $\widehat{H}_{\nu,q,n}(t, a)\in E^{\alpha}$. $\widehat{H}_{\nu,q,n}(t,a) < \Phi^{-1}(\alpha/2)$ means that market is experiencing a "negative" inefficiency, whereas $\widehat{H}_{\nu,q,n}(t,a) > \Phi^{-1}(1-\alpha/2)$ indicates that a "positive" inefficiency is occurring. Table 1summarizes how to characterize the behaviour of a financial time series depending on the value of $H(t)$.

Figure 2 gives an idea of how the estimator works with a simulated mBm, whereas Figure 3 displays the estimated $H(t)$ for the S&P500.

3. Empirical analyses

3.1. Data and preliminary analysis.

The methodology recalled in the previous section can be used to analyze the effects of COVID-19 pandemic on financial markets. Our study has considered the fifteen stock indexes summarized in table (2), whose daily closing indexes have been downloaded by the "Oxford-Man Institute's realized library", version 0.3 Gerd et al. (2009). For each index, the table reports the day of its major downturn and the corresponding pointwise Hölder exponent ($\widehat{H}_{\min}$), estimated by $\widehat{H}_{21,1,n}(t,(-1,1))$ at the p-level $\alpha = 0.05$ (n is the length of the data sample, varying with each examined time series). Notice that the reference day of $\widehat{H}_{\min}$ does not necessarily coincide with the low of the market. Thus, for example, while the second-worst day ever for the DJIA occurred on March 16 and three out of 15 worst days ever of the US market occurred between March 9 and 16, the minimal $\widehat{H}$ is reached on March 23 for both the S&P500 and the DJIA. As a

Table 1
Financial interpretation of $H(t)$.

$H(t)$	Stochastic effect	Investors' sentiment	Market effect
$> \Phi^{-1}\left(1-\frac{\alpha}{2}\right)$	Persistence Low variance	Future information will confirm past positions	Low volatility/Underreaction Overconfidence/Positive inefficiency
$\in E^{\alpha}$	Independence Martingale	Past information fully discounted by prices	Efficiency (at $(1-\alpha)$%))
$< \Phi^{-1}(\alpha/2)$	Mean-reversion High variance	Future information will contradict past positions	High volatility/Overreaction Negative inefficiency

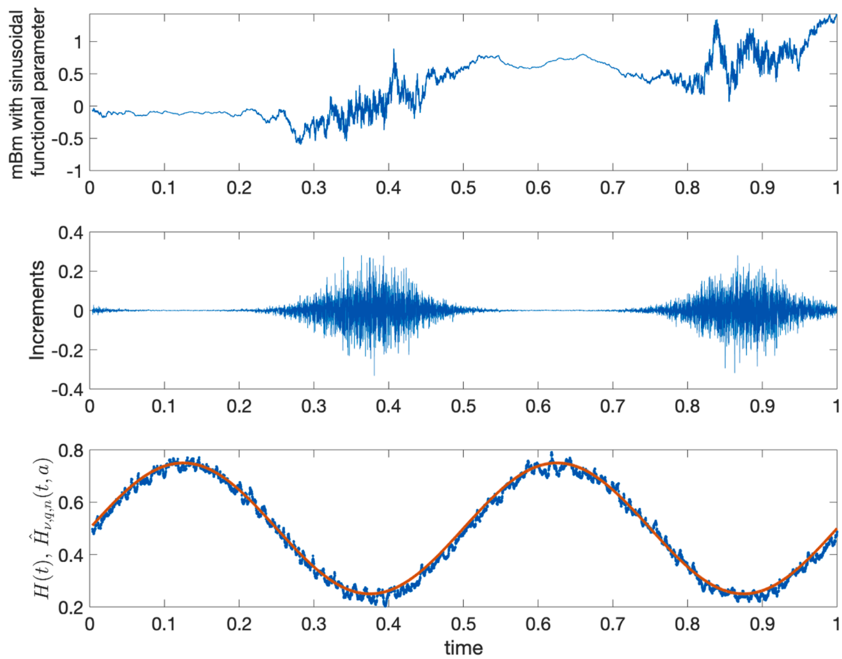


Fig. 2. Top panel: Surrogate mBm with functional parameter $H(t) = 0.5 + 0.25 * \sin(4 \cdot \pi \cdot t)$ with time support [0,1], Mid panel: Increment process $Y(t,a) = B_{H(t+a),K}(t+a) - B_{H(t),K}(t)$; Bottom panel: Functional parameter $H(t)$ (red continuous line) and estimates $\widehat{H}_{30,1,8192}(t,(-1,1))$ (dots).

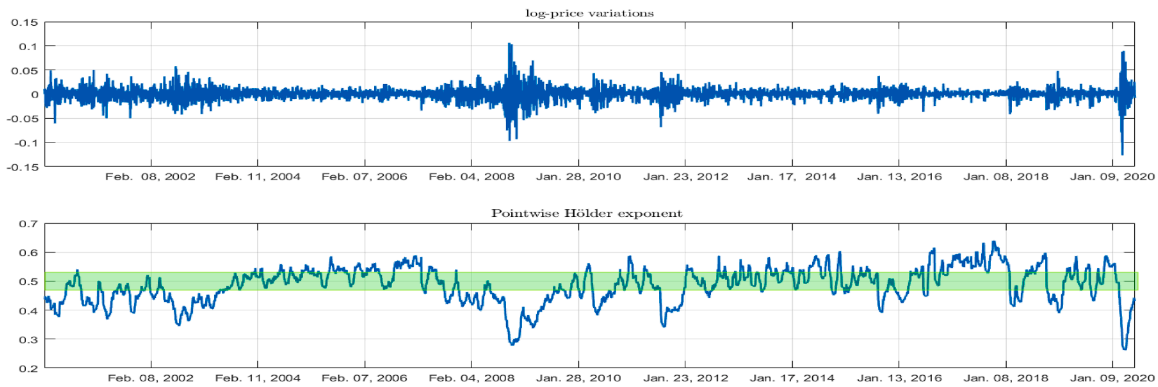


Fig. 3. Top panel: S&P500, log-index variations from January 3rd, 2000 to June 10th, 2020 (5127 daily stock quotes); Bottom panel: $\widehat{H}_{30,1,5127}(t,(-1,1))$, the area in green indicates E^{α}, with $\alpha = 0.05$. All times such that the corresponding $H(t)$ fall outside E^{α} indicate periods in which market experiences inefficiencies. The impact of COVID-19 crisis reflects in the large negative inefficiency that appears in the far-right.

term of comparison we use the 2007-2009 global systemic crisis. The last column of the table displays the difference ($\Delta\widehat{H}_{\min}$) between the estimated pointwise regularity of the COVID-19 crisis and of the 2007 – 2009 crisis. In term of $\widehat{H}_{\min}$, a slight difference can be appreciated for almost all stock indexes, except for HSI and N225, subject to a more severe burst of volatility in 2007 –2009 crisis with

Table 2
Stock indexes and major crashes.

Index	Symbol	2007 – 2009 crash		COVID-19 crash		
		Date	$\widehat{H}_{min}$	Date	$\widehat{H}_{min}$	$\Delta\widehat{H}_{min}$
All Ordinaries	AORD	Oct. 13, 2008	0.311	Mar. 23, 2020	0.295	-0.016
Bovespa	BVSP	Oct. 16, 2008	0.293	Mar. 23, 2020	0.270	-0.023
Cac 40	CAC	Oct. 17, 2008	0.291	Mar. 23, 2020	0.302	0.011
Dow Jones Ind Avg.	DJIA	Oct. 20, 2008	0.328	Mar. 23, 2020	0.296	-0.032
Footsie Mib	FTMIB	Oct. 20, 2008	0.293	Mar. 23, 2020	0.280	-0.013
Footsie 100	FTSE	Oct. 17, 2008	0.307	Mar. 18, 2020	0.320	0.013
Dax	GDAXI	Oct. 17, 2008	0.310	Mar. 23, 2020	0.328	0.018
Hang Seng	HSI	Oct. 20, 2008	0.338	Mar. 13, 2020	0.440	0.102
Ibex 35	IBEX	Oct. 17, 2008	0.299	Mar. 18, 2020	0.294	-0.005
Korea Composite	KOSPI	Oct. 24, 2008	0.299	Mar. 11, 2020	0.336	0.037
Ipc Mexico	MXX	Oct. 16, 2008	0.312	Mar. 23, 2020	0.347	0.035
Nikkei 225	N225	Oct. 24, 2008	0.293	Mar. 23, 2020	0.368	0.075
OMX Stockholm PI	OMXS	Oct. 20, 2008	0.296	Mar. 23, 2020	0.303	0.007
Standard & Poor's 500	S&P500	Oct. 20, 2008	0.278	Mar. 23, 2020	0.261	-0.017
Euro Stoxx 50	SX5E	Oct. 17, 2008	0.323	Mar. 13, 2020	0.333	0.010

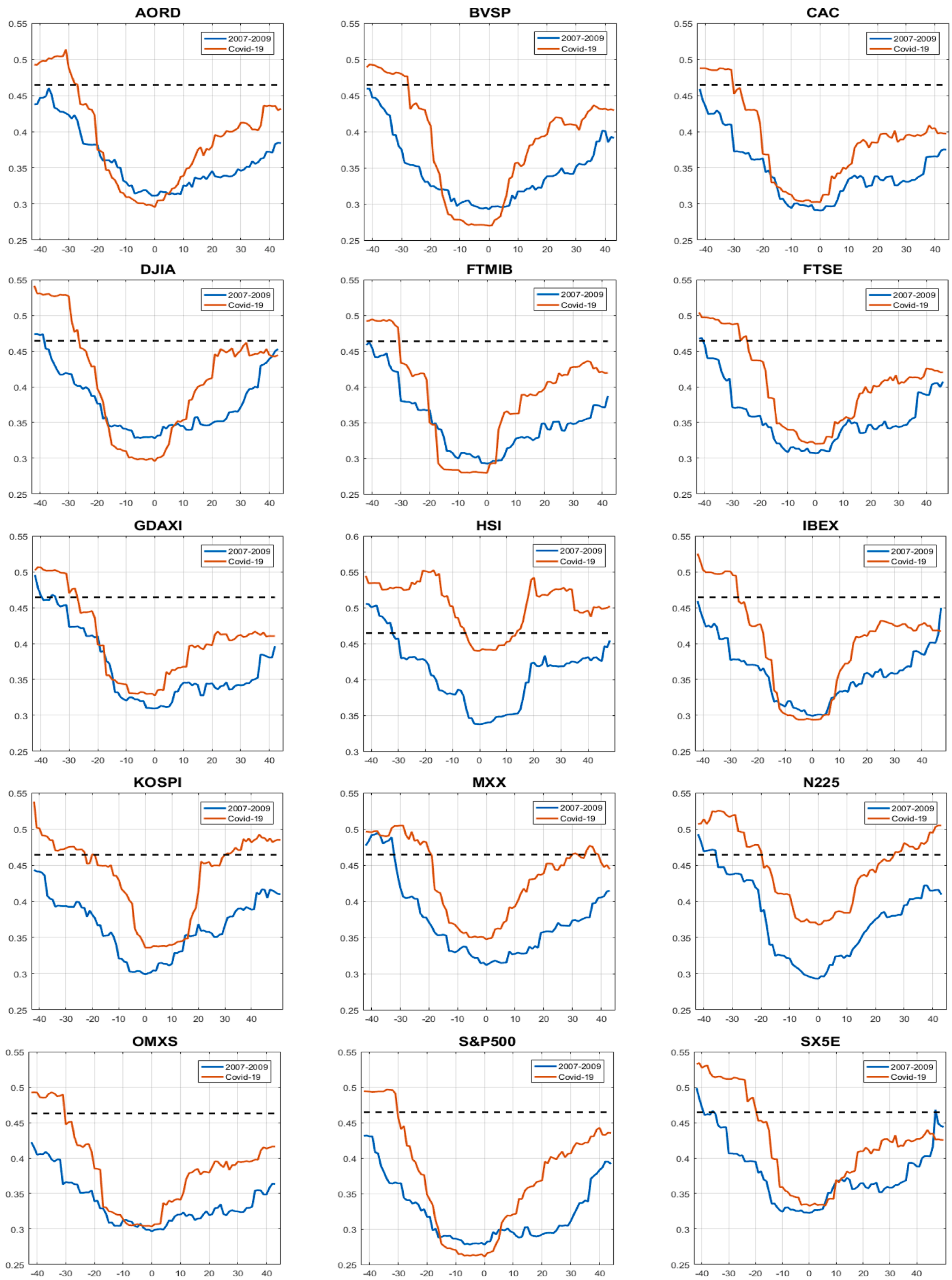
Table 3
Crash Index/Efficiency Index.

Index	2007 – 2009 crash									COVID-19 crash					
	D_{ex}	$\widehat{H}_{ex}$	CI^{+}	D_{re}	$\widehat{H}_{re}$	EI^{+}	D^{*}_{re}	$\widehat{H}^{*}_{re}$	EI^{+}	D_{ex}	$\widehat{H}_{ex}$	CI^{+}	D^{*}_{re}	$\widehat{H}^{*}_{re}$	EI^{+}
AORD	76	0.465	-2.0	200	0.467	0.8	44	0.384	1.7	27	0.468	-6.4	44	0.432	3.1
BVSP	105	0.474	-1.7	165	0.466	0.1	43	0.391	2.3	28	0.481	-7.5	43	0.429	3.7
CAC	90	0.466	-1.9	198	0.470	0.9	44	0.375	1.9	31	0.484	-5.9	44	0.397	2.2
DJIA	39	0.469	-3.6	48	0.462	2.8	43	0.453	2.9	27	0.476	-6.7	43	0.445	3.5
FTMIB	47	0.466	-3.7	219	0.471	0.8	42	0.387	2.2	31	0.480	-6.5	42	0.420	3.3
FTSE	41	0.469	-4.0	150	0.467	1.1	46	0.407	2.2	25	0.472	-6.1	46	0.420	2.2
GDAXI	36	0.468	-4.4	47	0.466	3.3	42	0.396	2.0	28	0.478	-5.4	42	0.410	2.0
HSI	33	0.480	-4.3	56	0.465	2.3	15*	0.358	1.7	6	0.472	-5.3	15*	0.468	1.9
IBEX	94	0.472	-1.8	151	0.466	1.1	47	0.450	3.2	28	0.495	-7.2	47	0.417	2.6
KOSPI	43	0.472	-4.0	126	0.470	1.4	31*	0.367	2.2	23	0.466	-5.7	31*	0.465	4.2
MXX	33	0.491	-5.4	157	0.467	0.1	30*	0.366	1.8	19	0.466	-6.3	30*	0.468	4.0
N225	36	0.473	-5.0	72	0.468	2.4	27*	0.380	3.2	20	0.471	-5.1	27*	0.470	3.8
OMXS	96	0.471	-1.8	218	0.471	0.8	43	0.364	1.6	31	0.489	-6.0	43	0.416	2.6
S&P500	107	0.465	-1.7	192	0.483	1.1	44	0.392	2.6	31	0.493	-7.5	44	0.435	4.0
SX5E	36	0.465	-3.9	47	0.469	3.1	47*	0.469	3.1	20	0.465	-6.6	47*	0.426	2.0

$^{+} \times 10^{-3}$.

Table 4
Monthly stock index correlation: 2007 – 2009/COVID-19 crash.

	Before crash		After crash	
(month)	(2)	(1)	(1)	(2)
AORD	0.88	0.93	0.92	0.78
BVSP	0.86	0.82	0.92	0.74
CAC	0.86	0.93	0.88	0.68
DJIA	0.82	0.94	0.65	0.20
FTMIB	0.92	0.84	0.86	0.35
FTSE	0.79	0.96	0.73	0.54
GDAXI	0.91	0.93	0.79	0.19
HSI	-0.24	0.91	0.98	-0.55
IBEX	0.76	0.96	0.95	-0.14
KOSPY	0.86	0.89	0.82	0.72
MXX	0.52	0.91	0.88	0.55
N225	0.72	0.94	0.95	0.83
OMXS	0.91	0.88	0.77	0.75
S&P500	0.83	0.93	0.32	0.90
S5XE	0.71	0.93	0.77	0.31



(caption on next page)

Fig. 4. Pointwise regularity dynamics before and after the major crashes of 2007 − 2009 global systemic crisis (blue line) and COVID-19 crisis (red line). Time 0 is the date of the major downturns recordered during the two crises. Notice the different patterns: while in 2007 − 2009 crisis the recovery was slow but somewhat univocal, in the case of the COVID-19 after an immediate recovery, the pointwise regularity of most European and US indexes has flattened below of the lower threshold of efficiency, what indicates that a subtle antipersistence characterizes US and Europe-based financial markets.

respect to the one experienced during the COVID-19. Figure 4 displays the dynamics of the estimated pointwise exponents before and after the crashes, for each stock index.

3.2. Analysis of results

The analysis highlights a well-defined pattern common to all the estimated pointwise Hölder exponent dynamics before and after the two considered crashes. As shown in table (3), for all indexes, the number of trading days (D_{ex}) between the departure from efficiency ($\widehat{H}_{\nu,q,n}(t,a) \leq \Phi^{-1}(\alpha/2)$) and the minimum regularity exponent is larger for the 2007 − 2009 crisis than for the COVID-19 scenario, and the time needed to return to the pre-crisis level of efficiency (D_{re}) is smaller for COVID-19 crash. These results are confirmed by the comparison between the crash index, defined as $CI = \frac{H_{min}-H_{ex}}{D_{ex}}$, and the efficiency index $EI = \frac{H_{re}-H_{min}}{D_{re}}$. CI and EI measure, respectively, the velocity the crash is reached with once the index starts experiencing inefficiency (down phase), and the velocity to return to efficiency since the crash (up phase). For the COVID-19, the former is lower while the latter is higher, compared to the 2007 −2009 crisis. Notice that D^*_{re} represents the number of trading days since the COVID-19 crash to efficiency, provided that $\widehat{H}_{\nu,q,n}(t,a)$ is back to belong to E^{α}; otherwise, D^*_{re} stands for the number of days since the COVID-19 crash to last day in the sample. At a first glance, both the timing and the magnitude of the pointwise regularity drops in the different markets could induce the conclusion - which seems to confirm the findings in Capelle-Blancard and Desroziers (2020) - that the COVID-19 pandemic triggered only a momentary crisis in international financial markets, with apparently small influence of country-specific factors on stock markets responses. Actually, if one looks at Figure 4, this is not exactly the case, for at least two main different market reactions can be distinguished corresponding to the US/European and Asian markets:

- US and European markets. After a sudden recovery of efficiency which lasted about one trading month mostly starting from March 23rd and which is marked by convex pointwise regularity dynamics, the concavity changes and most indexes display a flattening of the pointwise regularity exponent under the lower threshold of efficiency. According to the model, this indicates that volatility is still too high to be compatible with market efficiency. Interestingly, the flattening is more pronounced for European indexes and for the DJIA, whereas it is less evident for the S&P500. Following the timeline of the COVID-19 crisis, this behaviour can be explained with the joint effect of an immediate profit taking in the first month after the downturn, followed by a lack of confidence of the markets in a quick recovery of the world economy. This interpretation may find support in the increase of announcements forecasting a full recovery of the world economy only in the long run. For example, on April 14, the economic update of the International Monetary Fund (IMF) reported that COVID-19 would likely have a severe impact on economic activity and lead to a "Great Lockdown" recession (International Monetary Fund (IMF)). IMF forecasted a 3% contraction of the global economy, with all advanced economies expected to fall into recession and contract by 6.1% in 2020, and China and India expected to grow, respectively, at 1.2% and 1.9%, the slowest rates in decades.
- Asian markets (N225, HSI, KOSPI). Strangely enough, despite the COVID-19 outbreak can be traced in Asia, the pandemic seems to have impacted on these markets only because of the contagion triggered by the other markets. Indeed, not only their pointwise regularity exponents reached minima which are larger than those of US and European markets, but after the shock, these markets have sharply come back on pre-crisis levels, with values of $\widehat{H}(t)$ belonging the efficiency interval, as if the impact of the pandemic were totally discounted.

Table 4 can help to characterize the diversity of the COVID-19 and 2007 − 2009 crises. For each stock index, the table reports the correlations between the series of pointwise regularity exponents estimated for the 2007 − 2009 and COVID-19 crises. The correlations have been calculated one and two months before and after the crashes. We observe that one month before the two dynamics are strongly correlated (the minimum coefficient is 0.82) and continue to be correlated (with the expection of S&P500) also one month after the crash, what indicates that the pattern of the immediate response to a shock remains substantially the same even when circumstances are very different. Enlarging the outlook to two months, correlations before the crash (with the exeption a negative value for HSI) remain quite strong, while after the crash they decrease and become substantially different one from each other. This is a consequence of the flattening of regularity which occurs for the COVID-19 crisis and not for the 2007 − 2009 one, as one can clearly see in Figure 4. A possible explanation for the slowdown in the recover of market efficiency may come from two reasons:

- the different nature of aid interventions taken by political and financial institutions to face the two crises. During the period we are considering with respect to the 2007 − 2009 crisis, large liquidity injections stimulated financial markets directly, while the interventions planned so far for the COVID-19 have generally had a social more than a financial impact.

Table 5
Cross-Correlation matrix: 1-month.

						Before: 2007 − 2009 crash									
						Before: COVID-19 crash									
	AORD	BVSP	CAC	DJIA	FTMIB	FTSE	GDAXI	HSI	IBEX	KOSPI	MXX	N225	OMXS	S&P500	SX5E
AORD	-	0.95	0.97	0.97	0.97	0.98	0.97	0.96	0.97	0.88	0.94	0.96	0.95	0.97	0.97
	-	**0.93**	**0.97**	**0.96**	**0.87**	**0.97**	**0.96**	**0.98**	**0.95**	**0.97**	**0.97**	**0.95**	**0.95**	**0.96**	**0.96**
BVSP	0.93	-	0.94	0.98	0.93	0.94	0.95	0.91	0.96	0.91	0.97	0.95	0.92	0.98	0.94
	0.93	**-**	**0.94**	**0.98**	**0.91**	**0.92**	**0.94**	**0.89**	**0.96**	**0.87**	**0.96**	**0.90**	**0.93**	**0.98**	**0.94**
CAC	0.97	0.88	-	0.97	0.96	1.00	0.99	0.87	0.98	0.82	0.91	0.94	0.98	0.97	1.00
	0.97	**0.94**	**-**	**0.98**	**0.96**	**0.99**	**1.00**	**0.93**	**0.98**	**0.90**	**0.95**	**0.91**	**1.00**	**0.98**	**1.00**
DJIA	0.85	0.94	0.90	-	0.93	0.88	0.87	0.88	0.94	0.91	0.95	0.95	0.88	0.99	0.88
	0.96	**0.98**	**0.98**	**-**	**0.94**	**0.97**	**0.98**	**0.93**	**0.99**	**0.90**	**0.97**	**0.92**	**0.97**	**1.00**	**0.98**
FTMIB	0.93	0.91	0.96	0.96	-	0.94	0.96	0.86	0.94	0.84	0.88	0.96	0.93	0.97	0.95
	0.87	**0.91**	**0.96**	**0.94**	**-**	**0.93**	**0.96**	**0.79**	**0.97**	**0.76**	**0.87**	**0.78**	**0.97**	**0.94**	**0.96**
FTSE	0.97	0.93	1.00	0.97	0.95	-	0.99	0.86	0.98	0.82	0.90	0.95	0.97	0.96	0.99
	0.99	**0.97**	**1.00**	**0.98**	**0.98**	**-**	**1.00**	**0.95**	**0.98**	**0.88**	**0.95**	**0.93**	**0.99**	**0.98**	**1.00**
GDAXI	0.97	0.95	0.99	0.98	0.97	0.99	-	0.90	0.99	0.84	0.93	0.95	0.98	0.98	0.99
	0.96	**0.94**	**1.00**	**0.98**	**0.96**	**0.99**	**-**	**0.94**	**0.98**	**0.89**	**0.94**	**0.91**	**1.00**	**0.97**	**1.00**
HSI	0.80	0.85	0.85	0.91	0.86	0.84	0.88	-	0.88	0.94	0.92	0.88	0.88	0.88	0.82
	0.97	**0.95**	**0.95**	**0.93**	**0.95**	**0.96**	**0.96**	**-**	**0.96**	**0.97**	**0.97**	**0.98**	**0.96**	**0.96**	**0.95**
IBEX	0.97	0.96	0.98	0.97	0.95	0.98	0.99	0.90	-	0.86	0.94	0.96	0.97	0.97	0.98
	0.98	**0.98**	**0.99**	**1.00**	**0.98**	**0.98**	**0.99**	**0.94**	**-**	**0.88**	**0.96**	**0.92**	**0.99**	**1.00**	**0.99**
KOSPI	0.78	0.77	0.73	0.90	0.82	0.79	0.86	0.94	0.94	-	0.90	0.91	0.82	0.85	0.68
	0.91	**0.91**	**0.88**	**0.91**	**0.86**	**0.88**	**0.89**	**0.96**	**0.91**	**-**	**0.94**	**0.94**	**0.91**	**0.89**	**0.88**
MXX	0.90	0.97	0.93	0.96	0.92	0.92	0.95	0.94	0.95	0.93	-	0.91	0.93	0.95	0.92
	0.97	**0.96**	**0.95**	**0.97**	**0.87**	**0.90**	**0.94**	**0.94**	**0.94**	**0.92**	**-**	**0.92**	**0.93**	**0.96**	**0.95**
N225	0.86	0.79	0.82	0.93	0.92	0.86	0.89	0.83	0.89	0.92	0.80	-	0.75	0.91	0.79
	0.95	**0.90**	**0.91**	**0.92**	**0.78**	**0.94**	**0.91**	**0.96**	**0.88**	**0.93**	**0.92**	**-**	**0.89**	**0.91**	**0.90**
OMXS	0.93	0.92	0.97	0.94	0.92	0.97	0.97	0.88	0.97	0.82	0.90	0.91	-	0.94	0.96
	0.95	**0.93**	**1.00**	**0.97**	**0.97**	**0.99**	**1.00**	**0.90**	**0.98**	**0.87**	**0.93**	**0.89**	**-**	**0.97**	**1.00**
S&P500	0.87	0.97	0.93	0.99	0.88	0.90	0.97	0.83	0.94	0.87	0.93	0.94	0.88	-	0.91
	0.96	**0.98**	**0.98**	**1.00**	**0.94**	**0.96**	**0.97**	**0.92**	**0.99**	**0.89**	**0.96**	**0.91**	**0.97**	**-**	**0.98**
SX5E	0.98	0.94	1.00	0.96	0.96	0.99	0.99	0.86	0.98	0.80	0.90	0.94	0.98	0.97	-
	0.99	**0.98**	**1.00**	**0.99**	**0.99**	**1.00**	**1.00**	**0.95**	**0.99**	**0.88**	**0.96**	**0.94**	**0.99**	**0.99**	**-**
							After 2007 − 2009 crash								
							After COVID-19 crash								
	AORD	BVSP	CAC	DJIA	FTMIB	FTSE	GDAXI	HSI	IBEX	KOSPI	MXX	N225	OMXS	S&P500	SX5E
AORD	-	0.91	0.89	0.76	0.91	0.89	0.83	0.66	0.93	0.51	0.86	0.76	0.84	0.75	0.90
	-	**0.97**	**0.97**	**0.98**	**0.88**	**0.97**	**0.95**	**0.93**	**0.96**	**0.96**	**0.99**	**0.95**	**0.96**	**0.98**	**0.96**
BVSP	0.87	-	0.90	0.93	0.93	0.88	0.88	0.84	0.97	0.85	0.94	0.93	0.89	0.88	0.89
	0.97	**-**	**0.95**	**0.98**	**0.92**	**0.94**	**0.94**	**0.90**	**0.98**	**0.91**	**0.97**	**0.90**	**0.94**	**0.99**	**0.95**
CAC	0.88	0.89	-	0.86	0.93	0.99	0.97	0.55	0.92	0.62	0.72	0.82	0.99	0.87	0.99
	0.97	**0.95**	**-**	**0.97**	**0.95**	**0.98**	**0.99**	**0.94**	**0.98**	**0.93**	**0.96**	**0.91**	**0.99**	**0.97**	**1.00**
DJIA	0.60	0.76	0.78	-	0.65	0.72	0.86	0.54	0.73	0.51	0.70	0.61	0.74	0.88	0.75
	0.98	**0.98**	**0.97**	**-**	**0.92**	**0.98**	**0.97**	**0.92**	**0.99**	**0.93**	**0.99**	**0.95**	**0.97**	**1.00**	**0.97**
FTMIB	0.89	0.94	0.90	0.76	-	0.88	0.83	0.78	0.95	0.85	0.85	0.95	0.90	0.64	0.86
	0.91	**0.92**	**0.95**	**0.92**	**-**	**0.92**	**0.96**	**0.84**	**0.84**	**0.69**	**0.89**	**0.80**	**0.95**	**0.94**	**0.96**
FTSE	0.88	0.86	0.99	0.83	0.92	-	0.96	0.51	0.90	0.60	0.68	0.81	0.98	0.85	0.99
	0.91	**0.94**	**0.98**	**0.98**	**0.92**	**-**	**0.94**	**0.92**	**0.93**	**0.96**	**0.92**	**0.94**	**0.99**	**0.97**	**0.99**
GDAXI	0.82	0.87	0.97	0.92	0.88	0.96	-	0.55	0.88	0.57	0.73	0.77	0.97	0.93	0.97
	0.95	**0.94**	**0.99**	**0.97**	**0.92**	**0.98**	**-**	**0.93**	**0.91**	**0.92**	**0.95**	**0.90**	**0.99**	**0.97**	**1.00**
HSI	0.59	0.84	0.53	0.63	0.78	0.46	0.50	-	0.78	0.95	0.93	0.89	0.57	0.38	0.44
	0.78	**0.75**	**0.65**	**0.83**	**0.82**	**0.80**	**0.68**	**-**	**0.73**	**0.98**	**0.69**	**0.74**	**0.66**	**0.84**	**0.92**
IBEX	0.91	0.97	0.92	0.86	0.95	0.90	0.88	0.78	-	0.81	0.90	0.92	0.87	0.78	0.91
	0.96	**0.98**	**0.98**	**0.99**	**0.84**	**0.94**	**0.91**	**0.94**	**-**	**0.90**	**0.98**	**0.92**	**0.93**	**0.94**	**0.95**
KOSPI	0.73	0.90	0.54	0.50	0.90	0.48	0.38	0.97	0.89	-	0.93	0.96	0.65	0.30	0.35
	0.92	**0.85**	**0.85**	**0.86**	**0.75**	**0.82**	**0.84**	**0.97**	**0.87**	**-**	**0.91**	**0.80**	**0.84**	**0.85**	**0.84**
MXX	0.76	0.94	0.74	0.89	0.80	0.70	0.36	0.93	0.89	0.87	-	0.90	0.73	0.78	0.35
	0.99	**0.97**	**0.96**	**0.99**	**0.89**	**0.97**	**0.89**	**0.92**	**0.98**	**0.95**	**-**	**0.94**	**0.96**	**0.98**	**0.96**
N225	0.75	0.95	0.67	0.58	0.93	0.62	0.51	0.93	0.93	0.96	0.93	-	0.76	0.09	0.25
	0.95	**0.90**	**0.91**	**0.95**	**0.80**	**0.96**	**0.90**	**0.88**	**0.92**	**0.95**	**0.94**	**-**	**0.92**	**0.93**	**0.90**
OMXS	0.83	0.85	0.92	0.90	0.93	0.84	0.90	0.57	0.87	0.58	0.72	0.76	-	0.91	0.92
	0.96	**0.94**	**0.99**	**0.97**	**0.95**	**0.99**	**0.99**	**0.94**	**0.93**	**0.94**	**0.96**	**0.92**	**-**	**0.97**	**0.99**
S&P500	0.41	0.42	0.69	0.88	0.35	0.68	0.85	0.10	0.44	0.09	0.30	0.23	0.58	-	0.75
	0.98	**0.99**	**0.97**	**1.00**	**0.94**	**0.97**	**0.97**	**0.92**	**0.99**	**0.92**	**0.98**	**0.93**	**0.97**	**-**	**0.97**
SX5E	0.89	0.87	0.99	0.85	0.91	0.99	0.97	0.50	0.91	0.58	0.69	0.78	0.92	0.88	-
	0.97	**0.94**	**0.96**	**0.98**	**0.97**	**0.99**	**1.00**	**0.92**	**0.98**	**0.86**	**0.96**	**0.86**	**0.99**	**0.98**	**-**

Table 6
Cross-Correlation matrix: 2-month.

	AORD	BVSP	CAC	DJIA	FTMIB	FTSE	GDAXI	HSI	IBEX	KOSPI	MXX	N225	OMXS	S&P500	SX5E
Before: 2007 – 2009 crash / Before: COVID-19 crash															
AORD	-	0.95	0.97	0.94	0.97	0.98	0.91	0.96	0.96	0.91	0.90	0.95	0.97	0.94	0.87
	-	**0.93**	**0.98**	**0.98**	**0.94**	**0.98**	**0.98**	**-0.46**	**0.97**	**0.91**	**0.78**	**0.93**	**0.96**	**0.97**	**0.98**
BVSP	0.95	-	0.97	0.99	0.94	0.98	0.95	0.96	0.97	0.95	0.97	0.95	0.95	1.00	0.97
	0.93	**-**	**0.93**	**0.95**	**0.88**	**0.96**	**0.95**	**-0.61**	**0.94**	**0.84**	**0.77**	**0.90**	**0.94**	**0.95**	**0.93**
CAC	0.96	0.96	-	0.97	0.96	0.99	0.99	0.96	1.00	0.90	0.95	0.97	1.00	0.96	1.00
	0.98	**0.93**	**-**	**0.95**	**0.97**	**1.00**	**0.99**	**-0.45**	**0.99**	**0.90**	**0.72**	**0.88**	**0.98**	**0.97**	**0.97**
DJIA	0.83	0.99	0.97	-	0.95	0.97	0.96	0.94	0.96	0.91	0.98	0.94	0.96	1.00	0.97
	0.98	**0.95**	**0.95**	**-**	**0.92**	**0.96**	**0.96**	**-0.56**	**0.96**	**0.90**	**0.78**	**0.94**	**0.95**	**0.98**	**0.98**
FTMIB	0.97	0.89	0.95	0.95	-	0.95	0.94	0.96	0.94	0.88	0.95	0.94	0.94	0.95	0.95
	0.98	**0.96**	**1.00**	**0.96**	**-**	**0.96**	**0.97**	**-0.84**	**0.98**	**0.94**	**0.62**	**0.84**	**0.98**	**0.95**	**0.80**
FTSE	0.96	0.97	0.99	0.98	0.96	-	0.98	0.96	1.00	0.93	0.96	0.96	0.98	0.98	1.00
	0.93	**0.95**	**0.99**	**0.95**	**0.95**	**-**	**0.99**	**-0.93**	**0.89**	**0.89**	**0.55**	**0.78**	**0.99**	**0.98**	**0.94**
GDAXI	0.97	0.95	0.99	0.96	0.95	0.98	-	0.94	0.99	0.86	0.95	0.96	1.00	0.96	0.99
	0.98	**0.95**	**0.99**	**0.96**	**0.97**	**-**	**1.00**	**-0.52**	**0.99**	**0.95**	**0.70**	**0.90**	**0.99**	**0.97**	**0.97**
HSI	0.95	0.95	0.95	0.95	0.96	0.96	0.92	-	0.95	0.91	0.94	0.96	0.94	0.95	0.95
	-0.80	**-0.85**	**-0.72**	**-0.61**	**-0.71**	**-0.75**	**-0.76**	**-**	**-0.67**	**-0.35**	**-0.08**	**-0.62**	**-0.74**	**-0.74**	**-0.63**
IBEX	0.96	0.97	1.00	0.97	0.97	1.00	0.99	0.96	-	0.91	0.95	0.97	0.99	0.97	1.00
	0.94	**0.91**	**1.00**	**0.95**	**0.98**	**0.98**	**0.99**	**-0.93**	**-**	**0.91**	**0.50**	**0.81**	**1.00**	**1.00**	**0.95**
KOSPI	0.82	0.88	0.94	0.89	0.88	0.90	0.81	0.88	0.85	-	0.91	0.77	0.84	0.88	0.86
	-0.02	**0.66**	**0.91**	**0.84**	**0.88**	**0.91**	**0.84**	**0.37**	**0.90**	**-**	**0.31**	**0.24**	**0.75**	**0.88**	**0.82**
MXX	0.94	0.97	0.95	0.99	0.93	0.96	0.95	0.95	0.94	0.94	-	0.93	0.95	0.98	0.95
	0.78	**0.77**	**0.72**	**0.78**	**0.62**	**0.66**	**0.70**	**-0.76**	**0.69**	**0.52**	**-**	**0.72**	**0.67**	**0.74**	**0.77**
N225	0.95	0.86	0.95	0.90	0.90	0.92	0.95	0.93	0.96	0.77	0.86	-	0.95	0.88	0.95
	0.93	**0.90**	**0.88**	**0.94**	**0.84**	**0.89**	**0.90**	**-0.60**	**0.90**	**0.82**	**0.72**	**-**	**0.90**	**0.94**	**0.86**
OMXS	0.97	0.95	1.00	0.90	0.94	0.98	0.99	0.94	0.99	0.85	0.95	0.96	-	0.95	0.99
	0.96	**0.94**	**0.98**	**0.95**	**0.98**	**0.97**	**0.99**	**-0.62**	**1.00**	**0.94**	**0.67**	**0.90**	**-**	**0.98**	**0.96**
S&P500	0.95	0.99	0.96	1.00	0.95	0.97	0.95	0.94	0.96	0.90	0.98	0.93	0.96	-	0.97
	0.97	**0.95**	**0.97**	**0.98**	**0.95**	**0.97**	**0.97**	**-0.61**	**0.98**	**0.93**	**0.74**	**0.93**	**0.98**	**-**	**0.97**
SX5E	0.96	0.97	1.00	0.98	0.95	1.00	0.99	0.96	1.00	0.90	0.96	0.97	0.99	0.97	-
	0.75	**0.75**	**0.90**	**0.96**	**0.91**	**0.89**	**0.89**	**-0.63**	**0.94**	**0.82**	**0.25**	**0.49**	**0.90**	**0.93**	**-**
After 2007 – 2009 crash / **After COVID-19 crash**															
	AORD	BVSP	CAC	DJIA	FTMIB	FTSE	GDAXI	HSI	IBEX	KOSPI	MXX	N225	OMXS	S&P500	SX5E
AORD	-	0.85	0.31	0.87	0.65	0.70	0.63	0.83	0.63	0.60	0.78	0.75	0.51	0.84	0.39
	-	**0.52**	**0.73**	**0.40**	**0.49**	**0.78**	**0.27**	**-0.57**	**0.11**	**0.64**	**0.47**	**0.89**	**0.89**	**0.86**	**0.69**
BVSP	0.94	-	0.84	0.97	0.85	0.89	0.90	0.43	0.91	0.81	0.99	0.82	0.86	0.97	0.86
	0.52	**-**	**0.61**	**0.25**	**0.48**	**0.72**	**0.64**	**-0.38**	**0.05**	**0.57**	**0.54**	**0.60**	**0.70**	**0.76**	**0.73**
CAC	0.76	0.86	-	0.84	0.86	0.98	0.98	0.66	0.98	0.86	0.86	0.67	0.97	0.86	0.99
	0.73	**0.61**	**-**	**0.34**	**0.21**	**0.86**	**0.43**	**-0.31**	**0.35**	**0.40**	**0.37**	**0.58**	**0.90**	**0.67**	**0.97**
DJIA	0.94	0.87	0.93	-	0.90	0.94	0.95	0.69	0.93	0.89	0.96	0.95	0.90	1.00	0.93
	0.40	**0.25**	**0.34**	**-**	**0.40**	**0.35**	**0.41**	**-0.26**	**0.47**	**0.52**	**0.39**	**0.48**	**0.32**	**0.53**	**0.38**
FTMIB	0.87	0.85	0.89	0.86	-	0.89	0.89	0.73	0.89	0.85	0.90	0.77	0.86	0.86	0.88
	0.78	**0.72**	**0.86**	**0.35**	**-**	**0.38**	**0.44**	**-0.50**	**-0.30**	**0.93**	**0.79**	**0.71**	**0.38**	**0.69**	**0.54**
FTSE	0.82	0.90	1.00	0.89	0.87	-	1.00	0.64	0.99	0.89	0.90	0.78	0.98	0.91	0.98
	0.78	**0.77**	**0.87**	**0.74**	**0.69**	**-**	**0.71**	**-0.40**	**-0.04**	**0.74**	**0.75**	**0.74**	**0.87**	**0.84**	**0.92**
GDAXI	0.84	0.91	0.98	0.90	0.87	1.00	-	0.61	0.99	0.87	0.91	0.77	0.97	0.92	0.99
	0.27	**0.64**	**0.43**	**0.41**	**0.44**	**0.34**	**-**	**-0.32**	**0.31**	**0.46**	**0.45**	**0.29**	**0.40**	**0.41**	**0.51**
HSI	0.63	0.59	0.71	0.64	0.73	0.69	0.67	-	0.69	0.80	0.63	0.54	0.69	0.65	0.69
	-0.59	**-0.30**	**-0.25**	**-0.40**	**-0.67**	**-0.43**	**-0.32**	**-**	**-0.08**	**-0.69**	**-0.67**	**-0.66**	**-0.48**	**-0.54**	**-0.28**
IBEX	0.85	0.91	0.97	0.90	0.88	0.99	0.98	0.65	-	0.86	0.91	0.78	0.98	0.92	0.99
	0.57	**0.45**	**0.69**	**0.76**	**0.29**	**0.59**	**0.65**	**0.20**	**-**	**0.35**	**0.35**	**0.41**	**0.60**	**0.47**	**0.66**
KOSPI	0.87	0.84	0.86	0.89	0.88	0.93	0.91	0.83	0.91	-	0.92	0.91	0.92	0.89	0.91
	0.90	**0.77**	**0.71**	**0.82**	**0.86**	**0.83**	**0.67**	**-0.46**	**0.78**	**-**	**0.85**	**0.94**	**0.86**	**0.86**	**0.69**
MXX	0.94	0.99	0.80	0.96	0.73	0.87	0.88	0.45	0.89	0.84	-	0.87	0.85	0.95	0.82
	0.47	**0.54**	**0.37**	**0.39**	**0.79**	**0.71**	**0.45**	**-0.28**	**-0.10**	**0.84**	**-**	**0.77**	**0.49**	**0.68**	**0.48**
N225	0.94	0.84	0.94	0.95	0.88	0.95	0.96	0.65	0.94	0.91	0.93	-	0.93	0.95	0.94
	0.89	**0.60**	**0.58**	**0.48**	**0.71**	**0.72**	**0.29**	**-0.74**	**-0.02**	**0.84**	**0.77**	**-**	**0.81**	**0.93**	**0.62**
OMXS	0.80	0.88	0.99	0.79	0.86	0.98	0.97	0.69	0.98	0.87	0.88	0.81	-	0.84	0.98
	0.89	**0.70**	**0.90**	**0.32**	**0.38**	**0.87**	**0.40**	**-0.50**	**0.19**	**0.58**	**0.49**	**0.81**	**-**	**0.84**	**0.86**
S&P500	0.97	0.84	0.93	1.00	0.90	0.94	0.95	0.70	0.93	0.90	0.95	0.95	0.91	-	0.93
	0.86	**0.76**	**0.67**	**0.53**	**0.69**	**0.82**	**0.41**	**-0.68**	**-0.01**	**0.83**	**0.68**	**0.94**	**0.84**	**-**	**0.63**
SX5E	0.78	0.89	0.99	0.86	0.88	0.98	0.99	0.64	0.99	0.84	0.87	0.70	0.98	0.88	-
	0.77	**0.79**	**0.98**	**0.82**	**0.69**	**0.91**	**0.84**	**-0.28**	**0.78**	**0.70**	**0.72**	**0.69**	**0.92**	**0.80**	**-**

- during the period considered in this study for the 2007 − 2009 crisis the shock was perceived by markets as overcome, but this is not the case for the COVID-19, whose impact on the future financial scenarios is still uncertain, mainly owing to the fact that possible outbreaks are expected even for the next months.

It is well-known that during the crises, stocks markets are more cross-correlated than usual, see e.g. Liu et al. (2020) and references therein. The phenomenon can be studied also for the regularity point of view. Table 5 reports the cross-correlation matrices between the estimated pointwise Hölder exponent sequences, one-month before and after the crashes. Likewise, Table 6 reports the cross-correlation matrices between the estimated pointwise Hölder exponent sequences, two-months before and after the crashes. Notice that they are not symmetric because the correlation of stock index I_1 with stock index I_2 is calculated with respect to the date of the crash of I_1, which obviously can be different from the date of the crash of I_2. Matrices 5 and 6 confirm the pattern already observed from the individual dynamics of $\widehat{H}(t)$: before the crashes the correlations among the estimated pointwise exponents are generally higher than those observed after the crash, for both one and two months. Nonetheless, some differences deserve to be noted: for the COVID-19 the cross-correlations one month after the crash continue to be decidedly high, whereas they dramatically decrease (up to negative values for IBEX and HSI) over the horizon of two months. This is ascribable to the diverse behaviours already observed in relation to the change of concavity in the pointwise regularity curve which occurs about one month after the crash.

4. Conclusions and further developments

In this paper we have analyzed the effects of the COVID-19 pandemic on market efficiency of fifteen stock indexes using the pointwise regularity exponent as a measure of volatility. Under the parsimonious assumption that the dynamics of indexes can be modelled by a multifractional Brownian motion, the regularity exponent at time t directly expresses the degree of efficiency of the market at the same time t. Our findings suggest that following the pandemic only Asian markets have recovered full efficiency, while European and US markets - after an initial rebound - have not yet returned to the pre-crisis level of efficiency. The inefficiency that currently characterizes US and European financial markets is of negative type, that is it originates moderately high levels of volatility. The response to the shock triggered by COVID-19 pandemic is substantially different from the one observed in the last 2007 −2009 global financial crisis. This can be imputed to the different nature of the interventions taken by the political as well as financial institutions and by the fact that the uncertainty still predominates on future perspectives.

References

Alexander, C., 2001. Market models, a guide to financial data analysis. Chichester: Yohn Wiley & Sons.
Ayache, A., 2013. Continuous gaussian multifractional processes with random pointwise Hölder regularity. Journal of Theoretical Probability 26, 72–93.
Benassi, A., Bertrand, P., Cohen, S., Istas, J., 2000. Identification of the hurst index of a step fractional brownian motion. Statistical Inference for Stochastic Processes 3 (1-2), 101–111.
Benassi, A., Cohen, S., Istas, J., 1998. Identifying the multifractional function of a gaussian process. Statistics and Probability Letters 39, 337–345.
Benassi, A., Jaffard, S., Roux, D., 1997. Elliptic gaussian non random processes. Revista Mathematica Iberoamerican 13, 19–89.
Bianchi, S., 2005. Pathwise identification of the memory function of the multifractional brownian motion with application to finance. International Journal of Theoretical and Applied Finance 8, 255–281.
Bianchi, S., Pantanella, A., Pianese, A., 2013. Modeling stock prices by multifractional brownian motion: an improved estimation of the pointwise regularity. Quantitative Finance 13, 1317–1330.
Bianchi, S., Pianese, A., Frezza, M., 2020. A distribution-based method to gauge market liquidity through scale invariance between investment horizons. Applied Stochastic Models in Business and Industry 1–16. https://doi.org/10.1002/asmb.2531.
Bollerslev, T., 1986. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307–327.
Capelle-Blancard, G., Desroziers, A., 2020. The stock market is not the economy? insights from the COVID-19 crisis. Covid Economics: Vetted and Real-Time Papers. CEPR.
Chen, Z., Daigler, R.T., 2006. Persistence of volatility in futures markets. Future Markets 26, 571–594.
Coeurjolly, J.F., 2001. Estimating the parameters of a fractional brownian motion by discrete variations of its sample paths. Statistical Inference for Stochastic Processes 4 (2), 199–227.
Coeurjolly, J.F., 2005. Identification of multifractional brownian motion. Bernoulli 11 (6), 987–1008.
Dupire, B., 1994. Pricing with a smile. Risk 7, 18–20.
Embrechts, P., Maejima, M., 2002. Selfsimilar Processes, Princeton Series in Applied Mathematics. Princeton University Press.
Engle, R.F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica 50, 987–1007.
Engle, R.F., Patton, A.J., 2001. What good is a volatility model? Quantitative Finance 1, 237–245.
Fama, F., 1970. Efficient capital markets: A review of theory and empirical work. Journal of Finance 25, 383–417.
Fouque, J.P., Lorig, M.J., 2011. A fast mean-reverting correction to heston's stochastic volatility model. SIAM Journal of Financial Mathematics 2, 221–254.
Fouque, J.P., Papanicolaou, G., Sircar, K.R., 2000. Mean-reverting stochastic volatility. International Journal of Theoretical and Applied Finance 3, 101–142.
Francq, C., Zakoian, J.M., 2019. GARCH Models: Structure, Statistical Inference and Financial Applications, 2nd. Wiley.
Frezza, M., 2018. A fractal-based approach for modeling stock price variations. Chaos: An Interdisciplinary Journal of Nonlinear Science 28, 091102.
Garcin, M., 2017. Estimation of time-dependent hurst exponents with variational smoothing and application to forecasting foreign exchange rates. Physica A: Statistical Mechanics and its Applications 483 (Supplement C), 462–479.
Gerd, H., Lunde, A., Shephard, N., Sheppard, K., 2009. Oxford-Man Institute's realized library. Oxford-Man Institute, University of Oxford.
Heston, S.L., 1993. A closed-form solution for options with stochastic volatility, with application to bond and currency options. Review of Financial Study 6, 327–343.
Hull, J., White, A., 1990. Pricing interest-rate derivative securities. Review of Financial Study 3, 573–592.
International Monetary Fund (IMF), 2020. World economic outlook. April.
Istas, J., Lang, G., 1997. Variations quadratiques et estimation de l'exposant de Hölder local d'un processus gaussien. Ann. Inst. Henri Poincaré 33, 407–436.
Kent, J.T., Wood, A.T.A., 1997. Estimating the fractal dimension of a locally selfsimilar Gaussian process using increments. J. of the Royal Stat. Soc. Series B. 59 (3), 679–700.
Kolmogorov, A.N., 1940. Wienersche spiralen und einigeandere interessante kurven im hilbertschen raum. C.R. (Doklady) Acad. Sci. USSR (NS), pp. 115–118.
Lévy Véhel, J., Barriére, O., 2008. Local hölder regularity-based modeling of RR intervals. Proceedings 21th IEEE International Symposium on Computer-Based Medical Systems, pp. 75–80.

Liu, H.Y., Manzoor, A., Wang, C.Y., Zhang, L., Manzoor, Z., 2020. The COVID-19 outbreak and affected countries stock markets response. International Journal of Environmental Research and Public Health 17, 2800.

Lux, T., Segnon, M., 2018. Multifractal models in finance: Their origin, properties and applications. In: Chen, S.-H., Kaboudan, M., Du, Y.-R. (Eds.), The Oxford Handbook of Computational Economics and Finance. Oxford Handbooks.

Malkiel, B., Shiller, R., 2020. Does covid-19 prove the stock market is inefficient?Pairagraph, 04 May 4.

Mandelbrot, B., 1963. The variation of certain speculative prices. Journal of Business 36, 392–417.

Mandelbrot, B., Van Ness, J.W., 1968. Fractional brownian, motion fractional noise and application. SIAM Review 10, 422–437.

Patton, A.J., Sheppard, K., 2015. Good volatility, bad volatility: Signed jumps and the persistence of volatility. Review of Economics and Statistics 97, 683–697.

Péltier, R.F., Lévy Véhel, J., 1995. Multifractional brownian motion: Definition and preliminary results. Rapport de recherche INRIA 2645, 1–39.

Pianese, A., Bianchi, S., Palazzo, A.M., 2018. Fast and unbiased estimator of the time-dependent Hurst exponent. Chaos: An Interdisciplinary Journal of Nonlinear Science 28, 31102.

Rachev, S.T., Mittnik, S., 1993. Modeling asset returns with alternative stable distribution. Econometric Review 12, 261–330.

Rachev, S.T., Mittnik, S., 2000. Stable Paretian Models in Finance. Wiley, New York.

Reussa, A., Olivaresb, P., Secoc, S., Zagsta, R., 2016. Risk management and portfolio selection using α-stable regime switching models. Applied Mathematical Sciences 10 (12), 549–582.

Revuz, D., Yor, M., 1999. Continuous Martingales and Brownian Motion, 3rd. Springer, Berlin / Heidelberg, pp. 95–105.

Samorodnitsky, G., Taqqu, M.S., 1994. Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, New York. 1994