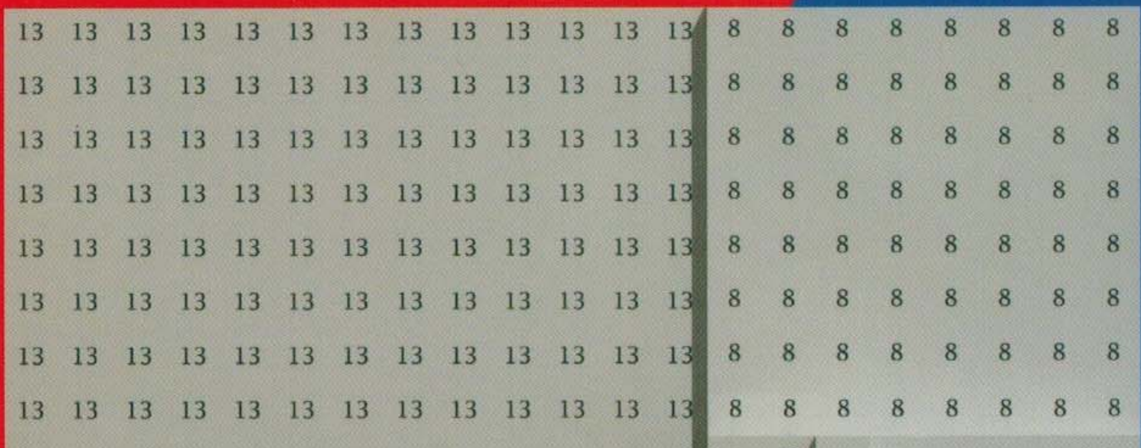
NEXUS 2018

Architecture and Mathematics

Conference Book



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11 – 14 June 2018, Pisa, Italy

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A Key Nexus for Vault Systems from Lecce: Stereometric Correlation Between Shape and Structure

Assunta Pelliccio,[1] *Marco Saccucci,*[2] *Ernesto Grande*[3]

Introduction

Vaulted surfaces have very ancient origins: already during the Mesopotamic age (tholos) or over the time in Mediterranean area (Sardinian *nuragheus*), it is possible referring to "pseudo-vaults" to indicate shells obtained through interaction among masonry units. In literature, these surfaces are often referred as "vault systems" due to the strong interaction between shape, geometry, material and structure. The link between the shape (signifer), often generated by complex geometries, and the structure (signified) of the different systems vault is in fact indivisible and always it strongly influences the space on which these systems insist.

The Research

Over the centuries, vault systems have generated different forms and constructive techniques depending on the development of geometric knowledge and the characteristics of local materials of the different geographic areas. During the 17th and 18th centuries this process, in the North of Italy, gave rise to the formation of the stellar and planetary vaults, in brickwork masonry, in Turin: these vaults, have a complex geometry due to the shapes generated by the composition of different surfaces (Spallone and Vitali 2017), getting movement and lightness effects. In the South, and particularly in the area around the city of Lecce, a new vault system, known as "Vault from Lecce", was developed (Gentile 1878).

In this area, the abundance of calcareous sand stone (the well-known Lecce stone, Cursi and Carparo stone), pliable and easy to work, together with the abilities of masters, have determined two different types of vault shells: "a squadro" (*square vault*) and "a spigolo" (*edge vault*). In this study the attention is focused on edge vaults because of the complexity of their geometry: in particular the surveyed vaults are located in Palmarigi, small town very close to Lecce.

In general, the architectural components of both the two types of vault are: the *columns*; the "*appese*" (*pendents*), which are the first rows of bricks having a section dimension greater than the section of columns; the "*formate*" (formed) namely the pointed arch; the *panels*; the *spherical cap* (Fig. 1a).

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Although these vaults are considered as a "refined transformation of the rib vault" (Colaianni 1967), the geometric genesis of their surface is really complex. Indeed, unlike rib vaults, the side groins of the four panels do not lie on surfaces developing along the diagonal planes (γ) with a common point on the vertical axis. They are in fact split along vertical surfaces (planes π-π') which result symmetrical with respect to the bisecting angle of the base, for both square and rectangular shape (Saccardi 2004) (Fig. 1b).

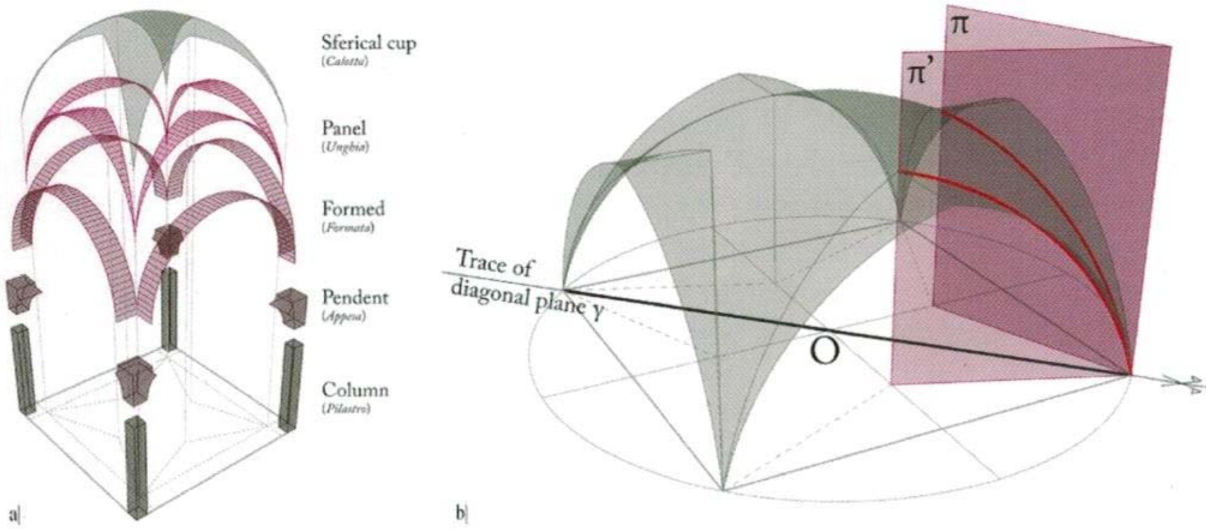

Fig. 1. a, left) Scheme of architectural components of vault from Lecce; b, right) Geometric genesis of vault from Lecce. Plane γ: diagonal plane of rib vault. Planes π-π': symmetrical vertical surfaces on which split the side groins of the four panels

The geometry parametrization of these vaults is particularly relevance in the current context of preservation of historical buildings and towns, where the use of information modelling systems (BIM) is a widespread strategy (Pelliccio et al. 2017). In particular, since modern software of structural analysis allow to derive the Finite Element model directly from the BIM, the reliability of the geometry of the model plays an important role.

In this paper the authors present a study concerning the geometry parametrization of the "Lecce vault systems". The results obtained from this study are used for deriving the FE model of a simple case in order to show the importance of the nexus between the geometry and the structure. From a geometric point of view, the "edge vaults" are usually characterized by a square bay (EV_SB) with a spherical cap or by a rectangular bay (EV_RB) with an ellipsoidal cap. They are obtained by the union of two surfaces S_1 and S_2, where: S_1 is a surface obtained by the intersection between a rib vault (with a semicircle or polycentric transverse arc) with a star-shaped polyhedron; S_2 is a surface obtained by the intersection between a spherical or ellipsoid shell with a star-shaped polyhedron.

Considering the geometric construction of the EV_SB, it is possible to observe that the union between the surfaces S_1 and S_2 leads to discontinuities if the transverse arch of rib vaults is a semicircle and the shell has spherical shape (Fig. 2a). Indeed, in this case the curvature (or "hump") of both the spherical cap and the panels have tangency points on the impost plane (A-B-C-D) and on the cusp (1'-2'-3'-4 ') only. On the contrary, these discontinuities do not appear when the transverse arch is an ogive with the profile coincident with the section obtained by the intersection of the spherical cap with the polyhedron.

In the case of the EV_RB, it is interesting to observe that the two transverse arches of rib vault have elliptic shape where the major axes are coincident with the sides of the basic rectangle, whilst the minor axes are coincident with the height of the key of the arc. In this case also, the intersection between the rib vaults (by elliptical transverse arches) and the ellipsoidal shell leads to a discontinuous surface (Fig. 2b). On the other hand, if the two profiles of the elliptical transverse arches, correspond to the section obtained by the intersection of the star shaped polyhedron with the ellipsoid, i.e., to an elliptical arch by double elliptical curvature, the obtained surface is continuous.

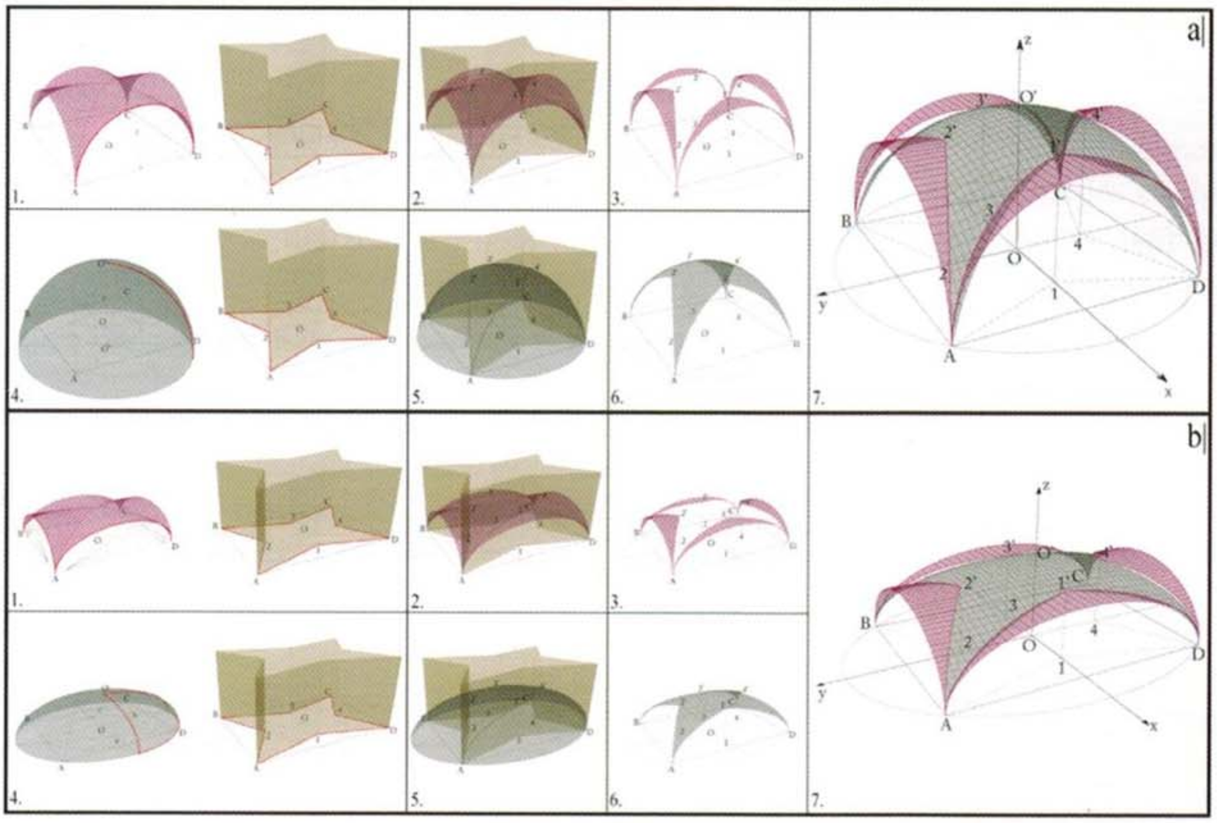

Fig. 2. Geometric construction of Edge vaults: a, aboe) Edge Vault on Square Bay (EV_SB); b, below) Edge Vault on Rectangular Bay (EV_RB)

From this analysis clearly emerges that the edge vaults with polycentric rib vault (ogive shape or double elliptical curvature transverse arch) allows to geometrically obtain continuous surfaces. On the contrary, the rib vault obtained by a semicircle or elliptic transverse arch are characterized by discontinuities. Then, while in the first case the finite element model of the edge vault can be directly derived from the rules at the basis of its geometric construction, in the second case it is not possible to obtain a consistent mesh for the structural model. In the latter case, it is necessary a 3D survey for deriving the real geometry.

Moreover, regarding the definition of the geometry of the EV_RB, the analysis has required the identification of the major ellipse on the plane xy, corresponding to the circumscribed ellipse to the rectangular base and the definition of the equation of ellipsoid passing through the points 1'-2'-3'-4 ' (points of cusps and common to both the shell and panel).

The major ellipse has to pass through the four vertices of the rectangle and it has to be concentric to the ellipse inscribed in the rectangle (Fig. 3a).

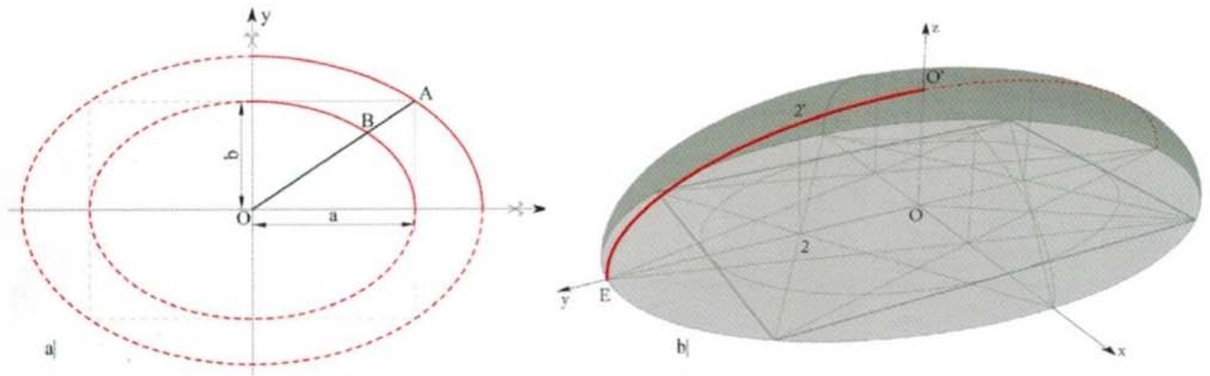

Fig. 3. a, left) Scale ratio between circumscribed and inscribed ellipses;
b, right) Construction of ellipsoid: ellipse on plane xz

At first, it is identified the scale ratio χ between the two ellipses (inscribed and circumscribed):

$$\chi = \frac{\overline{OA}}{\overline{OB}} \quad \text{(eq. 1)}$$

where the coordinates of the point B can be analytically obtained from the solution of the following system, where the eq. 2 refers to the ellipse and the eq. 3 to the diagonal of the rectangle:

$$\begin{cases} \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \\ y = \dfrac{b}{a}x \end{cases} \Rightarrow \begin{cases} x = \sqrt{\dfrac{a^2}{2}} & \text{(eq. 2)} \\ y = \sqrt{\dfrac{b^2}{2}} & \text{(eq. 3)} \end{cases}$$

Therefore, it results:

$$\overline{OA} = \sqrt{a^2 + b^2} \quad \text{(eq. 4)}$$

$$\overline{OB} = \sqrt{\left(\sqrt{\frac{a^2}{2}}\right)^2 + \left(\sqrt{\frac{b^2}{2}}\right)^2} = \sqrt{\frac{a^2 + b^2}{2}} \quad \text{(eq. 5)}$$

$$\chi = \frac{\overline{OA}}{\overline{OB}} = \frac{\sqrt{a^2 + b^2}}{\sqrt{\frac{a^2 + b^2}{2}}} = \sqrt{2} \qquad \text{(eq. 6)}$$

Once the major ellipse has been obtained, for the identification of the correct ellipsoid it is necessary to find the maximum height (O′) of the same ellipsoid through the definition of the minor ellipse on the plane xz (Fig. 3b).

In particular since the minor ellipse passing through points E-2' (2' is the known height of the point 2 obtained by intersection between the radius of star-shaped of polyhedron and the axis of the major ellipse), the height $\overline{OO'}$ can be defined due to the eq. 7.

$$\overline{OO'} = \sqrt{\left(\frac{x_E{}^2}{x_E{}^2 - x_{2'}{}^2}\right) y_{2'}{}^2} \qquad \text{(eq. 7)}$$

The geometry configuration of curved masonry structures (arches, vaults, domes) plays a crucial role on their structural behaviour. Indeed, the failure modes of this structures are generally characterized by the activation of global mechanisms particularly influenced by the geometry configuration.

Graphical and the numerical methods carried out for the structural analysis of vaults mainly account for this feature. For instance, the approaches proposed in the past by Mery and Heyman are based on the position of the thrust line with respect to the thickness of an arch derived from the vault (Fig. 4). The shape and the position of the thrust line, which furnishes information on the stability of the vault, are strictly dependent on the geometry of the arch, and then on the whole configuration of the vault.

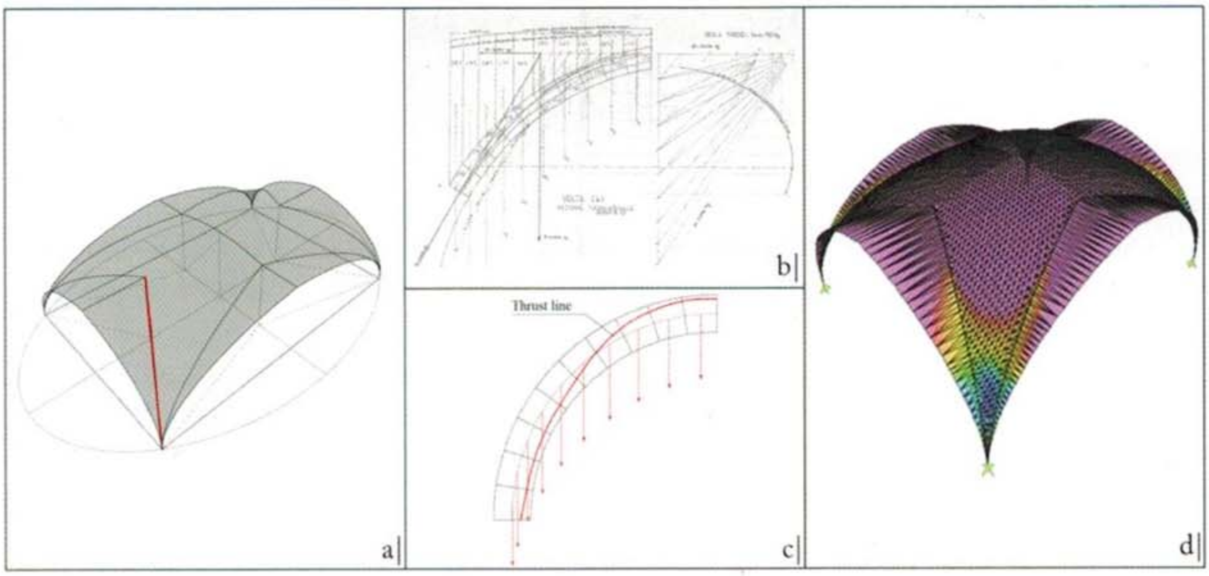

Fig. 4. Structural analysis: a) Edge vault on rectangular bay; b) and c) Graphical approach: Mery and Heyman methods; d) FEM analysis

The nexus between shape and structure assumes an important relevance also for modern approaches where the Finite Element Method allows to analyze the effective spatial behavior of vaults by using shell or solid elements (respectively assuming a thin behavior of vaults or considering the effect of thickness) together with advanced material constitutive laws (Romano and Grande 2008). The reliability of the results obtained by this method (Fig. 4d shows the map of principal stresses obtained from a 3D-FEM analyses of the edge vault) is strictly dependent on correctly considering both the geometry configuration of the model and structural details (arrangement of blocks, connections among the portions of vaults): both these aspects represent additional important information to consider in the context of the geometrical survey of masonry vaults.

Conclusion

In this paper the study concerning the analysis of Lecce vault systems has been presented with the aim to parameterize these surfaces. In particular, considering different types of Lecce vault systems, in some cases the rules and parameters at the basis of the union of two surfaces allow obtaining a continuous surface of the vault in agreement with its real geometry; in the others cases they lead to geometric discontinuities which do not characterize the real vault structure.

As underlined in the last part of the paper, the definition of the geometry of these complex structures is of particular interest in the derivation of reliable models for the structural analysis: here the nexus between the shape and the structure is testified by both the graphical methods of the past and modern numerical approaches.

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