WAVELIKE CORRELATIONS VERSUS PATH DETECTION: ANOTHER FORM OF COMPLEMENTARITY

Gennaro Auletta and **Gino Tarozzi**

Istituto di Filosofia
Università di Urbino, Italy
E–mail: g.tarozzi@uniurb.it

Received 16 June 2003; revised 6 October 2003

We show how quantum waves can produce EPR-type correlations and argue that the prediction of such wavelike correlations precludes the individuation of the path followed.

Key words: Wave, path detection, complementarity, entanglement, postselection.

1. INTRODUCTION

The two main foundational issues of quantum mechanics that have been discussed from the birth of the theory are, apart from the problem of measurement, EPR correlations and wave–particle duality. Here, we propose an experiment which could throw light on these features, which have always been the object of separate investigations, by showing the existence of a deep connection between them.

As is well known, wave-particle duality is a puzzle that has given rise to many different interpretations and proposals. Bohr tried to cast this problem in terms of a complementary relation between a wavelike and a corpuscular behaviour. However, many have interpreted this complementarity as if detection events represented the only reality [2, 6], so that the wave function would be only a mathematical tool or a representation of our knowledge [17]. Others have assigned a complete and simultaneous reality to both wave and particle [4]. In general, there have been many attempts at attributing ontological reality to the wave [5, 8] [9] [11]. In the present paper we propose a new way of considering the problem.

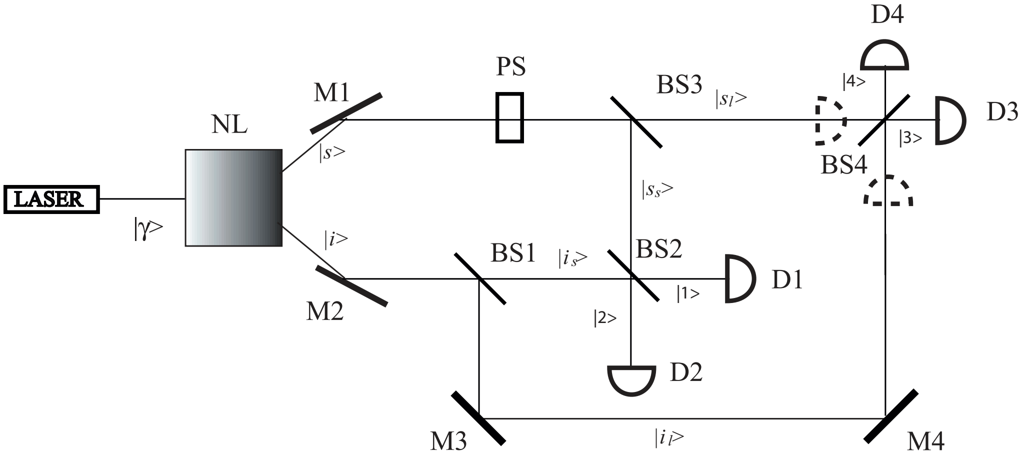

Fig. 1. A source laser pomps a photon in the state $|\gamma\rangle$. Successively, a parametric–down–conversion allows the emission of two photons, the *idler photon* (in the state $|i\rangle$) and *signal photon* (in the state $|s\rangle$). if γ has frequency ν and energy $h\nu$, the two outgoing photons have smaller frequencies ν_i and ν_s (and energies $h\nu_i$ and $h\nu_s$), respectively, with $\nu = \nu_i + \nu_s$. The two beam splitters BS1 and BS3 split each photon in two components, the 'shorter' and 'longer' components ($|i_s\rangle, |i_l\rangle$ and $|s_s\rangle, |s_l\rangle$, respectively). The two shorter (longer) components, $|i_s\rangle$ and $|s_s\rangle$ ($|i_l\rangle$ and $|s_l\rangle$) are recombined at BS2 (BS4), giving rise to photons in states $|1\rangle$ and $|2\rangle$ ($|3\rangle$ and $|4\rangle$), falling to detectors D1 and D2 (D3 and D4). Eventually, detectors D3 and D4 are placed before BS4 when D1 or D2 has already clicked.

We will firstly show the existence of a new property of quantum waves, namely that of producing correlations at-a-distance in a way surprisingly reminiscent of EPR states. Secondly, we will stress that such wavelike correlations are incompatible with the possibility of establishing which path was taken.

2. AN EXPERIMENTAL SET-UP

For this purpose, let us consider the following experimental setup. A laser (on the left) pumps a photon at-a-time into the non-linear crystal NL. The two outgoing photons (with the same frequency and in general in the same state) are called, as usual, *idler photon* (i) and *signal photon* (s). The s-photon goes through the beam splitter BS3, then it follows a shorter and a longer path. The i-photon undergoes an analogous transformation after it has gone through BS1. At BS2 and BS4 part of the i-photon and part of the s-photon are recombined and split again (finally reaching detectors D1 and D2, and D3 and D4). The beam splitters may be taken to be symmetric. In order to let the two components arrive together at BS4, one can easily accomodate a longer path of rectangular shape with 4 reflecting mirrors (not shown in the figure).

After the non-linear crystal and the first two beam splitters, the initial state evolves according to

$$|\gamma\rangle \overset{\text{NL,M1,M2}}{\longmapsto} \imath\,|s\rangle\,\imath\,|i\rangle \overset{\text{BS1,BS3}}{\longmapsto} -\frac{1}{2}\left[(\imath\,|s_s\rangle + |s_l\rangle)(|i_s\rangle + \imath\,|i_l\rangle)\right] \ , \tag{1}$$

where the subscripts s and l indicate the shorter and the longer path, respectively.

After the mirrors M3 and M4 the state evolves according to

$$\overset{\text{M3,M4}}{\longmapsto} -\frac{1}{2}\left[(\imath\,|s_s\rangle + |s_l\rangle)(|i_s\rangle - \imath\,|i_l\rangle)\right] \ . \tag{2}$$

After the two final beam splitters we obtain the final state

$$\overset{\text{BS2,BS4}}{\longmapsto} -\frac{1}{4}\left[\imath\,(\imath\,|1\rangle + |2\rangle) + (|3\rangle + \imath\,|4\rangle)\right]\left[(|1\rangle + \imath\,|2\rangle) - \imath\,(\imath\,|3\rangle + |4\rangle)\right] = |f\rangle \ . \tag{3}$$

It is easy to show that this state reduces to

$$|f\rangle = \frac{1}{4}\left(|1\rangle\,|1\rangle + |2\rangle\,|2\rangle - |3\rangle\,|3\rangle - |4\rangle\,|4\rangle - 2\imath\,|1\rangle\,|4\rangle - 2\imath\,|2\rangle\,|3\rangle\right) \ . \tag{4}$$

This state is a very interesting one. If we discard the cases in which both photons are detected by the same detector and only consider the case in which, for each photon pair, one is detected by one detector and the other by a different one (by performing a postselection), we obtain that, when detector D1 clicks, detector D4 must click as well, and when detector D2 clicks, detector D3 must also click. Here, we have a special kind of EPR state [3], which, in its standard normalized form, may be written as

$$|f'\rangle = \frac{1}{\sqrt{2}}\left(\imath\,|1\rangle\,|4\rangle + |2\rangle\,|3\rangle\right) \ . \tag{5}$$

It is interesting to stress that, because of this entanglement, we cannot know which photon has been detected by which detector, though, thanks to the entanglement, we can predict with certainty whether detector D3 or detector D4 will click once either detector D1 or detector D2 has clicked.

Let us now consider a variation of this setup realized through the displacement of detectors D3 and D4 to a position before BS4 once a photon has already been detected by D1 or D2 [7]. In this case, we can obtain information about which photon has been detected by D1 or D2 and which photon has been detected by D3 or D4. The point is that we obtain this information after an event has already occurred (detection by D1 or D2), unlike Wheeler's original proposal. It is interesting to stress that here, though we can know which photon has been detected by which detector and therefore the paths they follow, we cannot foresee whether detector D3 or detector D4 will reveal the photon after either detector D1 or detector D2 has clicked. This is because we no longer produce the entanglement of the two photons by displacing detectors D3 and D4. Let us consider the state of the two photons after they have passed BS2 but before they pass BS4:

$$\begin{aligned} &\overset{\text{BS2}}{\mapsto} -\frac{1}{2\sqrt{2}}\left[\imath\left(\imath\left|1\right\rangle + \left|2\right\rangle\right) + \left|s_l\right\rangle\right]\left[\left(\left|1\right\rangle + \imath\left|2\right\rangle\right) - \imath\left|i_l\right\rangle\right] \qquad (6) \\ &= \frac{1}{2\sqrt{2}}\left[\left|1\right\rangle\left|1\right\rangle + \left|2\right\rangle\left|2\right\rangle - \left(\left|s_l\right\rangle + \imath\left|i_l\right\rangle\right)\left|1\right\rangle - \left(\imath\left|s_l\right\rangle + \left|i_l\right\rangle\right)\left|2\right\rangle + \imath\left|s_l\right\rangle\left|i_l\right\rangle\right] \ . \end{aligned}$$

It is easy to see that, if D1 clicks, the photon has the same probability of taking path $\left|s_l\right\rangle$ or $\left|i_l\right\rangle$, *i.e.,* D3 or D4 will click with equal probability; and the same is true if D2 clicks. Therefore, though BS2 already generates a wavelike behaviour, this is not sufficient to produce entanglement. The photons can be entangled only if both BS2 and BS4 work as proper beam splitters and we perform a suitable postselection.

It is also interesting to observe that, on account of the first interference (by BS2) and of the superposition of the two components of the i–photon and of the superposition of the two components of the s–photons, the latter situation (when detectors D3 and D4 are placed before BS4) is not at all the classical situation that would arise if both BS2 and BS4 were removed. In this case, if D1 clicks, we know with certainty that the i–photon has been detected and that the s–photon (if not detected by D2) will reach D3. On the other hand, if D2 clicks, we know with certainty that the s–photon has been detected and that the i–photon (if not detected by D1) will reach D4. We see that the conditional probabilities of the 'classical' case are neither similar to the conditional probabilities of the path detection after the interference at BS2, nor to the 'entangled' situation. In fact, in the latter case, if D1 clicks only once, D3 will click too, whereas in the 'classical' setup, if D1 clicks (and not D2), D3 will click as well; and similarly if D2 clicks.

3. COMPLEMENTARITY BETWEEN "ENTANGLED WAVES" AND PATH DETECTION

Summing up and leaving aside the less interesting 'classical' situation, we have here a new form of complementarity between the existence of correlations and the possibility of predicting the second detection event (by eliminating the case when both photons are revealed by the same detector), on the one hand, and corpuscolar behaviour with knowledge of the path but without the possibility of predicting the second detection event, on the other. The important feature is that we have a close connection between perfect correlation and wavelike behavior as well as between the absence of correlations and corpuscular behaviour. We believe that this connection may have significant consequences for our understanding of entanglement and complementarity. This is not to say that entanglement cannot be produced with particles, but rather that, in the context of this experiment, entanglement is associated with wavelike behaviour. We also think that this form of duality may be framed within Bohr's idea of complementarity [1]. Indeed, from the Como Congress in 1927 Bohr saw wave–particle duality as a consequence of a complementarity between mutually exclusive features, i.e. the causal requirement and the spatio–temporal individuation. This is also the case in our proposed experiment: On the one hand the causal wavelike dynamics (together with a suitable postselection) allows us to predict whether D3 *or* D4 will click, on the other the measurement of the position of the photons by detectors D3-D4 allows retrodiction about the path followed to D1 and D2 as well.

This discussion has made it clear that our proposed experiment may support a realistic interpretation of wave–particle duality because it seems to us that there is no reason to attribute reality only to the particle and not to the wave, since both aspects give rise to different and complementary predictions. Moreover, as we know, there are infinite intermediate cases between undulatory and corpuscular behavior [10, 12], and this supports our conclusion[1]. This procedure can also be applied to our experiment by varying the reflection and transmission parameters of BS1 and BS3. However, as already emerged in previously proposed (and abopve quoted) experiments, we like to stress that the 'superposed' or entangled, wavelike reality cannot be directly detected: If it could, this would mean that we would be able to measure the state of a quantum system with a single measurement act [15]. We can only infer it indirectly (by predicting where the second photon will go according to whether D1 or D2 clicks). This is precisely because any measurement is local in nature.

Aknowledgments. We thank Prof. Vincenzo Fano and Dr. Mauro Fortunato for fruitful discussion about these issues.

REFERENCES

1. Niels Bohr, "The quantum postulate and the recent development of atomic theory," *Nature* **121** (1928), 580–90.
2. Werner Heisenberg, *The Physical Principles of Quantum Theory* (University Press, Chicago, 1930).

---

[1] See also [16, part VII].

3. A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?," *Phys. Rev.* **47** (1935), 777-80.
4. Louis de Broglie, *Une tentative d'Interpretation Causale et Non-linéaire de la Mécanique ondulatoire* (Gauthier-Villars, Paris, 1956).
5. Franco Selleri, "On the wave function of quantum mechanics," *Lett. Nuovo Cimento* **1** (1969), 908–910.
6. Werner Heisenberg, "Remarks on the origin of the relations of uncertainty," in W. C. Price, and S. S. Chissick, eds., *The Uncertainty Principle and Foundations of Quantum Mechanics* (Wiley, New York, 1977), pp. 3–6.
7. John A. Wheeler, "The 'past' and the 'delayed-choice' double-slit experiment," in A. R. Marlow, ed., *Mathematical Foundations of Quantum Theory* (Academic, New York, 1978), pp. 9–48.
8. Franco Selleri, "On the direct observability of quantum waves," *Found. Phys.* **12** (1982), 1087–112.
9. Gino Tarozzi, "Experimental tests of the properties of the quantum-mechanical wave function," *Lett. Nuovo Cimento* **42** (1985), 438–42.
10. P. Mittelstaedt, A. Prieur, and R. Schieder, "Unsharp particle-wave duality in a photon split-beam experiment," *Found. Phys.* **17** (1987), 891–903.
11. Evandro Agazzi, "Waves, particles, and complementarity," in G. Tarozzi and A. van der Merwe, eds., *The Nature of Quantum Paradoxes* (Kluwer, Dordrecht, 1988), pp. 53–74.
12. D. M. Greenberger and A. Yasin, "Simultaneous wave and particle knowledge in a neutron interferometer," *Phys. Lett. A* **128** (1988), 391–94.
13. L. J. Wang, X. Y. Zou, and L. Mandel, "Experimental test of the Broglie guided–wave theory for photons," *Phys. Rev. Lett.* **66** (1991), 1111-14.
14. X. Y. Zou, T. P. Grayson, L. J. Wang, and L. Mandel, "Can an empty de Broglie pilot wave induce coherence?," *Phys. Rev. Lett.* **68** (1992), 3667–69.
15. G. M. D'Ariano and H. P.Yuen, "Impossibility of measuring the wave function of a single quantum system," *Phys. Rev. Lett.* **76** (1996), 2832–35.
16. Gennaro Auletta, *Foundations and Interpretation of Quantum Mechanics. In the Light of a Critical-Historical Analysis of the Problems and of a Synthesis of the Results* (World Scientific, Singapore, 2000; rev. edn. 2001).
17. Anton Zeilinger, "Why the quantum? It From bit? A partecipatory universe?: Three far–reaching visionary questions from John Archibald Wheeler and how they inspired a quantum experimentalist," in J. D. Barrow, P. C. W. Davies, and C. L. Harper, Jr.

eds., *Science and Ultimate Reality: Quantum Theory, Cosmology and Complexity* (Cambridge University Press, Cambridge, 2003)