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# Closed-form Solution for NSM Strengthening Systems Applied to Brittle Substrates

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**Abstract.** The article presents the results obtained by using an analytical model herein slightly modified to study Near Surface Mounted (NSM) CFRP strengthening applied to brittle supports. To this scope, three sets of lab investigations comprising CFRPs applied to concrete and masonry pillars were considered. The lab investigations comprised testing of different CFRP strips characterized by various geometrical (cross section and bonding length) and mechanical parameters (elastic modulus). The interfacial tau-slip laws adopted in the analytical model were tuned using available experimental data and idealized bilinear laws proposed by the authors in the experimental campaigns. The reliability of the model was assessed by comparing the global force-slip curves obtained experimentally and analytically. Also, the analytical model allowed estimating the axial stress in the FRP strengthening, providing valuable information about the most probable failure mode (i.e., debonding or fiber failure). A global satisfactory agreement was found in all the set of tests studied with the proposed model. The analytical approach was able to accurately estimate the initial stiffness, peak load and post peak behavior.

## INTRODUCTION

Near Surface Mounted (NSM) fiber reinforced polymeric strengthening materials consist of inserting FRP strips or bars into grooves cut into masonry or concrete panels [1]. Compared to externally bonded FRPs, NSM presents important advantages such as minimum invasiveness of the intervention, better aesthetics, and less susceptibility to environmental aggressiveness, just to cite a few. Similarly to externally bonded (EB) FRPs, the strengthening material is bonded to the support through adhesive layers which are entailed to ensure the final performance of the strengthening intervention. Thus, the adhesive-to-substrate and the FRP-to-adhesive bond performance are critical parameters to consider during the design of such strengthening interventions. The bond quality resulted deeply influenced by different parameters including the mechanical properties of the support, the adhesive type and the geometry of the FRP strengthening. Several studies were devoted to study the role played by the aforementioned parameters on the final performance of NSM FRP strengthening solutions [1-4]. The technical literature is abundant in terms of experimental investigations developed taking into account concrete supports [3] while less attention was paid to masonry substrates [1,2,4]. In case of concrete supports, NSM CFRP materials showed different failure modes including debonding at laminate-adhesive interface or mixed failure modes combining debonding and cohesive failures depending on the type of adhesive adopted (flexible or stiff). This finding was also reflected in the observed cracking patterns which resulted characterized by localized damage in the adhesive for flexible ones, while stiff

adhesive showed to foster the propagation of damages to the concrete support as well [3]. The application of NSM FRP strips on masonry supports confirmed the development of intermediate crack debonding failures involving the progressive detachment of the FRP strip. Also, different parameters were studied experimentally to understand their influence on FRP-to masonry joint bonding performance [1,2,4]. Considering the various similarities between EB and NSM in terms of failure modes, reduction of the problem to a bond loss event and the possibility of concentrating all non-linearities in the inelastic interfaces between the FRPs and the support, this work explores the possibility of adopting an analytical model developed by the authors [5] and previously validated with respect to EB FRPs to analyze NSM CFRP solutions applied to brittle supports. The analytical model is herein slightly modified and adopted to analyze a total of three sets of pull-out tests of NSM CFRPs from concrete and masonry supports. The reliability of the model is assessed with respect to global force-slip curves adopting the interfacial bond curves obtained from available experimental data or idealized bilinear laws proposed by the authors. Also, the model is able to provide important information related to the axial stress level in the FRP strips. Thus, allowing the fast and reliable evaluation of different failure modes involving debonding of the strengthening and/or fiber failures.

## ANALYTICAL MODEL

This section summarizes the analytical approach already published by the authors in [5] and herein slightly modified to demonstrate its validity to study Near Surface Mounted (NSM) strengthening techniques applied to brittle supports (i.e., masonry and concrete prisms). For the sake of brevity, only the main assumptions and final closed form solution will be discussed in what follows. The equations at the basis of the proposed approach are derived from equilibrium considerations involving an infinitesimal zone of the FRP and specifically considering a debonding mechanism propagating at the strengthening/support interface. The following three assumptions were adopted: (i) the strengthening material behaves as elastic during the whole loading process, (ii) all the nonlinearities concentrate at the interface between FRP strip and masonry support and (iii) the FRP-to-support interface is associated only with a Mode II tangential fracture ruled by a  $\tau(x)$ - $s(x)$  curve. In the present work, a two-branches  $\tau(s) - s(x)$  relationship is considered: branch 1 (or Phase 1) is characterized by a linear phase until the peak tangential stress  $f_b$  is reached (see Eq. 1); branch 2 (or Phase 2) is defined by a nonlinear softening exponential law (see Eq. 2):

$$\tau(s) = ks(x) \quad (1)$$

$$\tau(s) = f_b \cdot e^{-\frac{(s-s_*)f_b}{G_{II}}} = \tau_0 \cdot e^{-\frac{s f_b}{G_{II}}}, \text{ with } \tau_0 = f_b \cdot e^{\frac{s_* f_b}{G_{II}}} \quad (2)$$

where:  $k$  is the slope of the linear branch;  $f_b$  identifies the peak tangential strength at the interface;  $G_{II}$  stands for the rate of fracture energy associated with Mode II in the post-peak stage;  $s_*$  is the slip value at the end of the elastic branch. The selected shear stress-slip law for the interface allows to identify three possible cases for the behavior of the specimen during debonding mechanisms: (i) Case 1, holds for the entire bonding length  $L_L$  which behaves as elastic (Eq. 3); (ii) Case 2 is characterized by a mixed interface response (Eq. 4); Case 3, characterized by the entire bonding length behaving in Phase 2 (Eq. 5). The closed-form solutions obtained considering each Case is reported below in terms of the normal stress in the FRP strengthening.

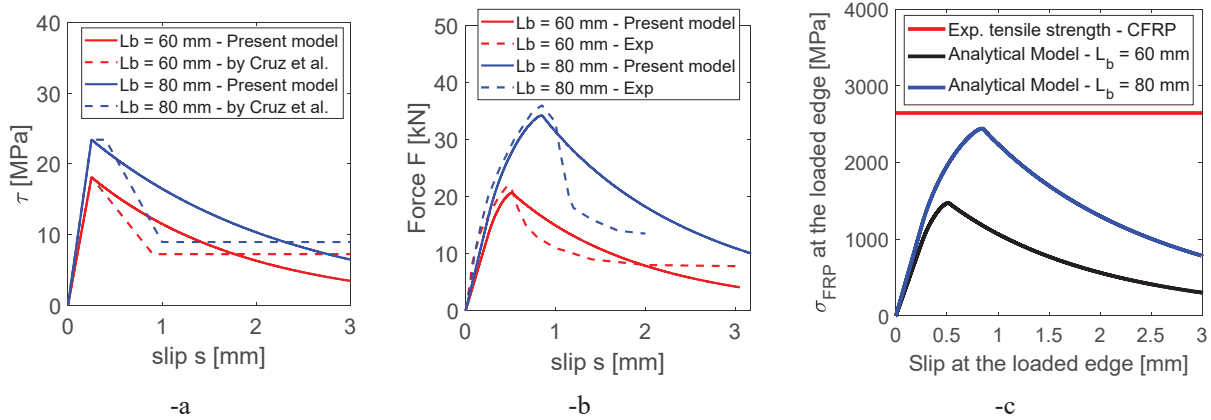
$$\text{Case 1} \quad \sigma_{FRP} = E_{FRP} \cdot \frac{ds(x)}{dx} = E_{FRP} \cdot \alpha \cdot \frac{s_0}{2} \cdot (e^{\alpha x} - e^{-\alpha x}) \quad (4)$$

$$\text{Case 2} \quad \sigma_{FRP} = \sqrt{\frac{2\tau_0 \cdot G_{II} \cdot E_{FRP}}{t_{FRP} \cdot f_b} \cdot \left( C_1 - e^{-\frac{s(x) \cdot f_b}{G_{II}}} \right)} \quad (5)$$

$$\sigma_{FRP}^L = \sqrt{\frac{2\tau_0 \cdot G_{II} \cdot E_{FRP}}{t_{FRP} \cdot f_b} \cdot \left( C_1 - e^{-\frac{s_L f_b}{G_{II}}} \right)} \quad (6)$$

## ANALYTICAL INVESTIGATION

The proposed model is used to analyze two sets of lab investigations comprising NSM retrofitting of concrete and masonry pillars tested by Cruz and coworkers in [3] and by Kashyap and coworkers in [4], respectively. Several concrete cubes were reinforced with 60 (adhesive 1) and 80 (adhesive 2) mm long CFRP NSM strengthening with cross section equal to 10x1.4 mm and subjected to pull out tests. The CFRP strip was assumed to behave as an elastic material during the simulation having Elastic Modulus equal to 169.5 GPa and peak tensile strength of 2648.3 MPa. A comparison between the bond-slip curve adopted in the present work and the one suggested by the authors in [3] is presented in FIGURE 1-a. FIGURE 1-b presents the mean pullout force versus loaded end slip curve provided in [3] compared with the results obtained using the proposed analytical model, while FIGURE 1-c shows the tensile stress in the CFRP obtained with the proposed model and compared to the experimental value.



**FIGURE 1.** Comparison between experimental and analytical results: tau-slip curves (-a), force-slip curves (-b) and tensile stress in the FRP NSM strip (-c).

The comparison depicted in FIGURE 1-b shows a good agreement between the experimental results and analytical outcomes in terms of initial elastic phase, peak load and post peak behavior. From the experimental point of view, the failure mode was characterized by debonding at CFRP-adhesive interface for 60 mm long CFRP strip, while the concrete cubes reinforced with 80 mm long CFRP were subjected to fiber failures. Even if the present model is not able to take into account this failure mode, the analysis of the tensile stress evolution during the pull-out in the CFRP strip shown in FIGURE 1-c clearly evidences the arising of high tensile stress in the FRP material, in agreement with the experimental outcomes. The second benchmark was carried out by considering a set of masonry pillars reinforced with CFRP NSM strips tested by Kashyap and co-workers [4]. The lab tests comprised the pull-out testing of masonry pillars reinforced with NSM CFRP strips having cross section equal to 10x3.6 mm<sup>2</sup>. The NSM strengthening was bonded considering a bonding length equal to 420 mm. Finally, the mechanical parameters of the CFRP strengthening, as reported in [4], were: Elastic modulus of 165 GPa and tensile strength equal to 2.7 GPa. The tangential stress versus slip curve adopted in the analytical model was tuned on the experimental one provided by the authors [4] (see FIGURE 2-a). FIGURE 2-b shows the comparison between the analytical and experimental results. As clearly visible, the model is able to accurately predict the initial elastic phase, while it provides a slightly lower peak load with respect to the experimental envelope.

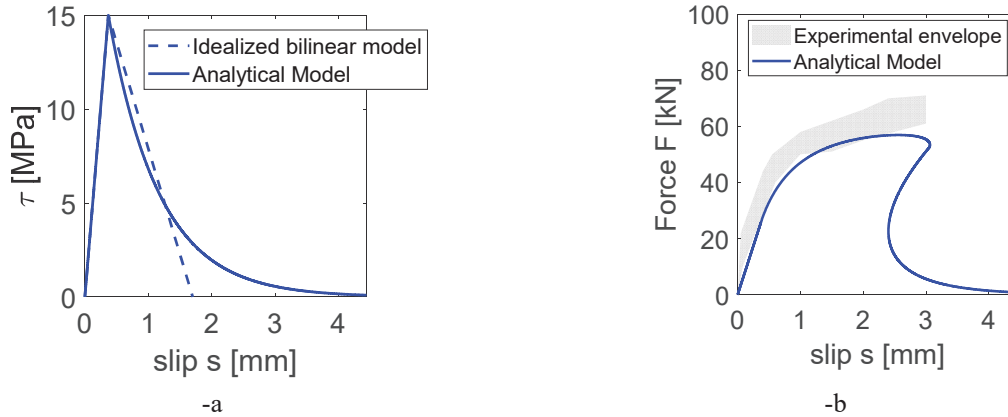


FIGURE 2. Numerical results obtained with: (-a) the FEM model and (-b) the spring model.

## CONCLUSIONS

The work presents the results obtained using an analytical model developed by the authors and already validated with respect to externally bonded FRP strips subjected to single-lap shear tests and used to reinforce brittle substrates, to analyze NSM CFRP reinforced masonry and concrete prisms subjected to pull out tests. In detail, the interface tau-slip laws adopted were calibrated on the basis of the available experimental data or idealized bilinear laws provided by the authors. The reliability of the analytical approach was evaluated mainly by comparing the experimentally and analytically obtained global force-slip curves. A good agreement was found in terms of initial stiffness, peak load and post peak behavior considering both supports and all series of test studied. Furthermore, even if the model was not able to capture failure modes involving fiber failure, it allowed to estimate the axial stress evolution in the CFRP reinforcement, providing valuable information on the expected failure mode.

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