The Centrodes Relevance for the Kineto-Dynamic Analysis of the Parabolic Rigid-Body Motion

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Abstract. This paper is aimed to show the relevance of the centrodes for the kineto-dynamic analysis of the parabolic rigid-body motion in the vertical plane. In particular, the instant center of rotation and the acceleration pole are determined along with their corresponding vector fields. Consequently, the fixed and moving centrodes are achieved in order to reproduce and simulate the parabolic rigid-body motion by means of their pure-rolling contact. The inflection circle is also derived by kinematic properties only, since the acceleration pole coincides with the inflection pole, which is opposite to the instant center of rotation. A suitable algorithm has been formulated and validated by significant examples.

Keywords: kineto-dynamic analysis · vector fileds · centrodes · inflection circle

1 Introduction

The kinematic analysis of mechanical systems plays an important role in the engineering field, enabling designers and researchers to understand, predict, and optimize the motion of objects and systems [1–3]. A very interesting aspect is the one related to the kineto-dynamics, which refers to understanding how forces affect the movement of objects. This field is important for designing and analyzing everything from machines to vehicles, helping the designers to figure out how they behave when they are in motion [4–6]. The planar kinematics of a rigid body can be traced back to the determination of the centrodes, since by means of the pure rolling of the moving centrode on the fixed centrode it is possible to obtain the exact law of motion of the body considered. Hence the need to have analytical tools available that allow to calculate the positions assumed by the instant center of rotation during motion [7–9]. The use of geometric loci, as the centrodes, together with the inflection circle is essential in kinematic analysis, offering a deeper understanding of instantaneous motion and relative motion between the bodies that compose the whole system. They are used to analyze the motion of mechanisms, helping engineers to design efficient and reliable mechanical systems [10–14].

This paper is aimed to show the relevance of the centrodes for the kineto-dynamic analysis of the parabolic rigid-body motion in the vertical plane. In particular, the IC of rotation, the acceleration pole, centrodes and inflection circle are determined. A suitable algorithm has been formulated and validated by significant examples.

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2 Kineto-Dynamic Analysis and Centrodes

The kineto-dynamic analysis of the parabolic rigid-body motion in the vertical plane for different input data is formulated by supposing to launch a rigid link by means of the pendular motion of the human arm.

Thus, referring to Fig. 1, the height h of the shoulder center C of a human being in upright position on the origin O of the fixed frame OXY and the length R_b of the human arm are both assigned as anatomical features. The X-axis is horizontal.

Moreover, the initial conditions are also assigned in terms of angular position θ_0 of the human arm, which is considered to be measured positive in counter-clockwise direction with respect to the Y-axis, and speed v_0 of the mass center G of the rigid link AB of length L that is supposed to be aligned with the extended arm and having G coincident with the hand center. Consequently, one has

$$x_0 = R_b \sin\theta_0 \quad y_0 = h - R_b \sin\theta_0 \tag{1}$$

$$v_{0x} = v_0 \cos\theta_0 \quad v_{0y} = v_0 \sin\theta_0 \quad \omega_0 = v_0/R_b \tag{2}$$

where x_0 and y_0 are the Cartesian coordinates of G at the initial position G_0, while v_{0x}, v_{0y} and ω_0 are the components of the velocity vector $\mathbf{v}_0$ and the angular velocity, respectively, at the same initial position.

The position analysis is developed by referring to the following position vectors of points G, A and B respectively:

$$\mathbf{r}_G = \left[\, v_{0x}t + x_0 \;\; v_{0y}t - \tfrac{1}{2}gt^2 + y_0 \;\; 1 \,\right]^T \tag{3}$$

$$\mathbf{r}_A = \left[\, x_G + \tfrac{L}{2}\cos(\omega_0 t + \theta_0) \;\; y_G + \tfrac{L}{2}\sin(\omega_0 t + \theta_0) \;\; 1 \,\right]^T \tag{4}$$

$$\mathbf{r}_B = \left[\, x_G - \tfrac{L}{2}\cos(\omega_0 t + \theta_0) \;\; y_G - \tfrac{L}{2}\sin(\omega_0 t + \theta_0) \;\; 1 \,\right]^T \tag{5}$$

where g and t are the acceleration gravity and time, respectively.

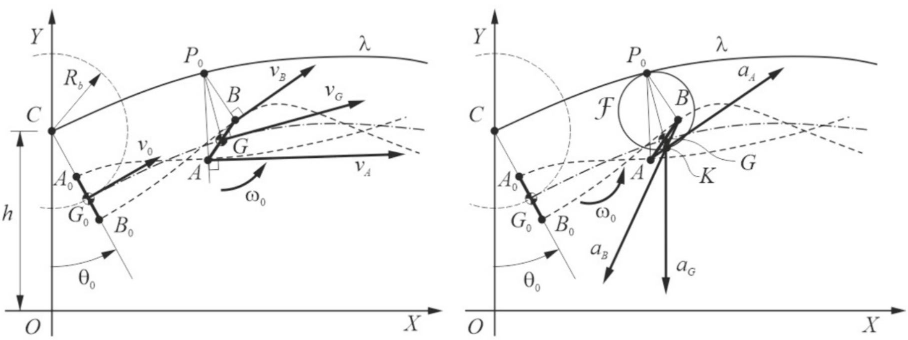

Fig. 1. Kinematic analysis: a) velocity; b) acceleration.

The velocity vectors $\dot{\mathbf{r}}_G$, $\dot{\mathbf{r}}_A$ and $\dot{\mathbf{r}}_B$ of points G, A and B, respectively, are obtained by developing the first time-derivative of Eqs. (3), (4) and (5), as follows

$$\dot{\mathbf{r}}_G = \left[v_{0x} \ v_{0y} - gt \ 1 \right]^T \tag{6}$$

$$\dot{\mathbf{r}}_A = \left[\dot{x}_G - \omega_0 \tfrac{L}{2} \sin(\omega_0 t + \theta_0) \ \dot{y}_G + \omega_0 \tfrac{L}{2} \cos(\omega_0 t + \theta_0) \ 1 \right]^T \tag{7}$$

$$\dot{\mathbf{r}}_B = \left[\dot{x}_G + \omega_0 \tfrac{L}{2} \sin(\omega_0 t + \theta_0) \ \dot{y}_G - \omega_0 \tfrac{L}{2} \cos(\omega_0 t + \theta_0) \ 1 \right]^T \tag{8}$$

Similarly, developing the second time-derivative of Eqs. (3), (4) and (5), one has

$$\ddot{\mathbf{r}}_G = \left[0 \ -g \ 1 \right]^T \tag{9}$$

$$\ddot{\mathbf{r}}_A = \left[\ddot{x}_G - \tfrac{L}{2} \omega_0^2 \cos(\omega_0 t + \theta_0) \ \ddot{y}_G - \tfrac{L}{2} \omega_0^2 \sin(\omega_0 t + \theta_0) \ 1 \right]^T \tag{10}$$

$$\ddot{\mathbf{r}}_B = \left[\ddot{x}_G + \tfrac{L}{2} \omega_0^2 \cos(\omega_0 t + \theta_0) \ \ddot{y}_G + \tfrac{L}{2} \omega_0^2 \sin(\omega_0 t + \theta_0) \ 1 \right]^T \tag{11}$$

Referring to Fig. 2, the position vector $\mathbf{r}_{P0}$ of the instant center of rotation P_0 is now obtained as intersecting point according to the Chasles theorem, by the knowledge of the velocity vectors of points A and G.

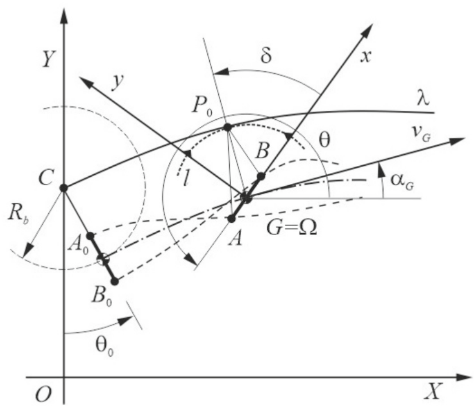

Fig. 2. Fixed λ and moving l centrodes.

Thus, it takes the form

$$\mathbf{r}_{P0} = \mathbf{r}_\lambda = \frac{1}{\dot{x}_G \dot{y}_A - \dot{x}_A \dot{y}_G} \begin{bmatrix} x_G \dot{x}_G \dot{y}_A - x_A \dot{x}_A \dot{y}_G + \dot{y}_G \dot{y}_A (y_G - y_A) \\ -y_A \dot{x}_G \dot{y}_A + y_G \dot{y}_A \dot{y}_G + \dot{x}_G \dot{x}_A (x_G - x_A) \\ 1 \end{bmatrix} \tag{12}$$

Equation (12) expresses also the fixed centrode λ since representing the position vector of P_0 with respect to the fixed frame OXY.

The moving centrode l is expressed with respect the moving frame $\Omega\ x\ y$ that is attached to the rigid link AB in the following form

$$\mathbf{r}_l = \sqrt{(x_{P0} - x_G)^2 + (y_{P0} - y_G)^2} \left[\cos\delta \ \sin\delta 1 \right]^T \tag{13}$$

where $\delta = 270° - (\theta - \alpha_G)$ with $\alpha_G = \tan^{-1}(\dot{y}_G / \dot{x}_G)$.

In order to simulate the pure-rolling motion of the centrodes, the transformation matrix $\mathbf{T}$ between the rigid link AB and the fixed frame OXY is derived as follows

$$\mathbf{T} = \begin{bmatrix} \cos\theta & -\sin\theta & x_\Omega \\ \sin\theta & \cos\theta & y_\Omega \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The equation of the inflection circle F that passes through the inflection pole W that coincides with the acceleration pole K, and the instant center P_0, is determining by intersecting the accelerations vectors $\ddot{\mathbf{r}}_G$, $\ddot{\mathbf{r}}_A$ and $\ddot{\mathbf{r}}_B$ of points G, A and B, respectively.

Thus, the position vector $\mathbf{r}_W = \mathbf{r}_K$ of both points W and K is given by

$$\mathbf{r}_W = \mathbf{r}_K = \left[x_G \ y_A + \frac{\ddot{y}_A}{\ddot{x}_A}(x_G - x_A) \ 1 \right]^T \tag{15}$$

In fact, being ω_0 constant, all acceleration vectors are directed toward K and, as a consequence, the acceleration pole coincides with the inflection pole.

3 Graphical and Numerical Examples

The proposed formulation has been implemented in Matlab and validated by means of significant examples for different input data. In particular, the graphical results of Fig. 3a, b and c are obtained for the following input data: $h = 1.5$ u, $R_b = 0.65$ u, $v_0 = 5$ u/s, $L = AB = 0.4$ u, when $\theta = 45°$, while the graphical results of Fig. 3d and e are obtained for $h = 1.5$ u, $R_b = 0.65$ u, $v_0 = 4$ u/s, $L = AB = 0.4$ u, when $\theta = 90°$.

The velocity and acceleration vector fields are shown in Fig. 3a and d, along with the fixed centrode and the inflection circle, while Fig. 3b, c and e show both fixed and moving centrodes during their pure-rolling motion.

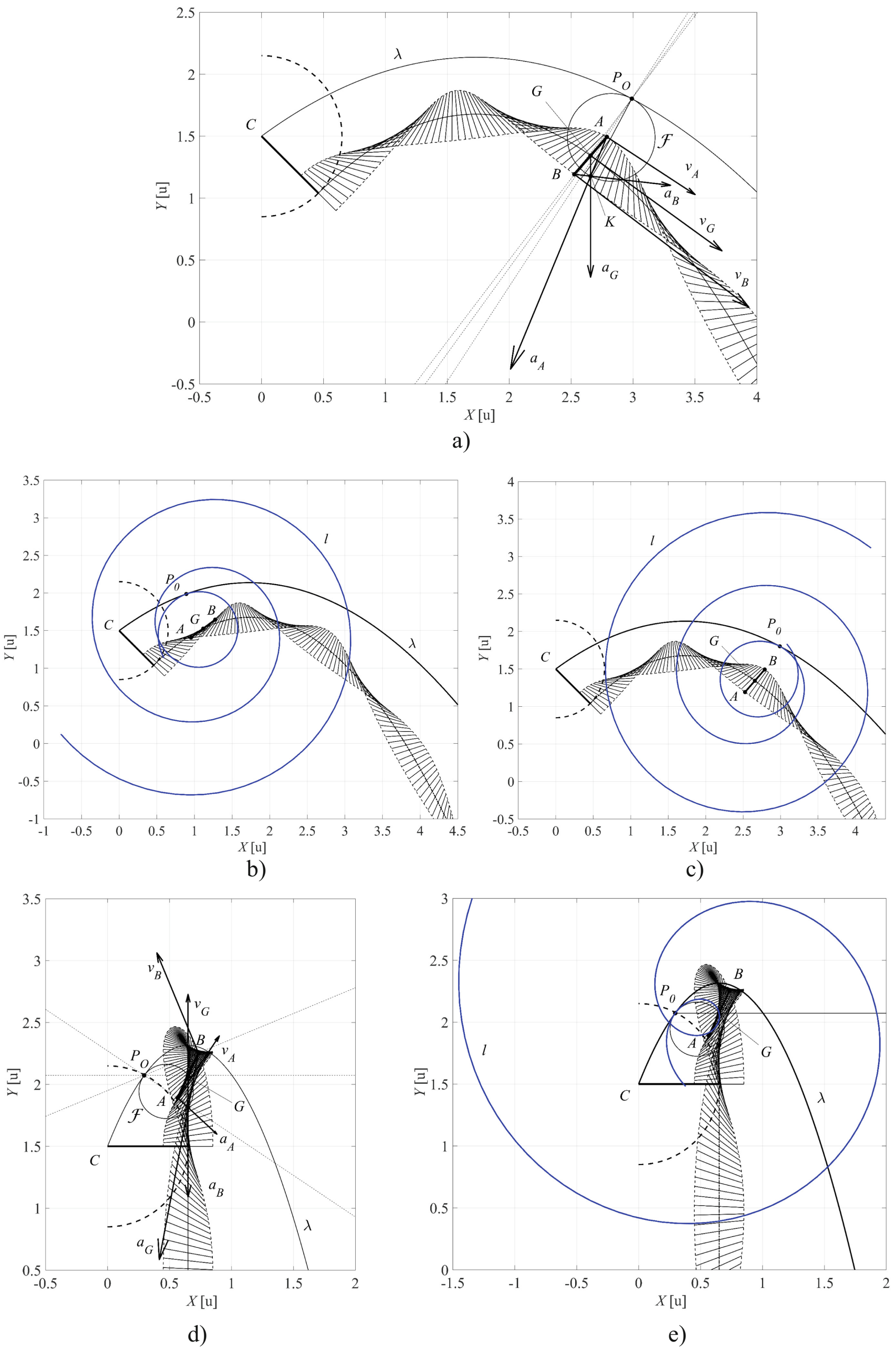

Fig. 3. Case 1 for $\theta = 45°$: a) velocity and acceleration vector fields; b) & c) centrodes; Case 2 for $\theta = 90°$: d) velocity and acceleration vector fields; e) centrodes.

4 Conclusions

The relevance of the centrodes for the kineto-dynamic analysis of the parabolic rigid-body motion in the vertical plane has been investigated and put in evidence.

In particular, the instant center of rotation, acceleration pole, centrodes and inflection circle have been determined. A suitable algorithm has been formulated with the aim to show and analyze this planar motion with the aid of the velocity and acceleration vector fields and by means of the centrodes pure-rolling motion, which has been validated through significant examples for different input data.

References

1. Pennestrì, E., Cera, M.: Engineering Kinematics: Curvature Theory of Plane Motion. Amazon, Roma (2023)
2. Norton, R.L.: Design of Machinery an Introduction to the Synthesis and Analysis of Mechanisms and Machines. McGraw-Hill Education, New York, N.Y. (2020)
3. Figliolini, G., Lanni, C.: Meccanica Applicata alle Macchine: applicazioni di dinamica dei sistemi meccanici. Società Editrice Esculapio, Bologna (2023). (in italian)
4. Di Gregorio, R.: Kinematics and dynamics of planar mechanisms reinterpreted in rigid-body's configuration space. Meccanica **51**(4), 993–1005 (2017)
5. Field, E.J., Armor, J.B., Rossitto, K.F., Mitchell, D.G.: Effects of pitch instantaneous center of rotation location on flying qualities. In: AIAA Atmospheric Flight Mechanics Conference and Exhibit, 5–8 August 2002, Monterey, California, AIAA 2002-4799. (2002)
6. Balike, K.P., Rakheja, S., Stiharu, I.: Kineto-dynamic performance analysis of vehicle with an asymmetric suspension damper using a roll-plane model. Int. J. Vehicle Perform. (IJVP) **1**(1), 1–23 (2013)
7. Schiller, R.W.: Method for determining the instantaneous center of acceleration for a given link. J. Appl. Mech. **32**(1), 217–218 (1965)
8. Meyer Zur Capellen, W., Dittrich, G.: Note on the determination of accelerations in plane kinematics. J. Mech. **1** (3), 315–319 (1966)
9. Gilmore, B.J., Cipra, R.J.: An analytical method for computing the instant centers, centrodes, inflection circles, and centers of curvature of the centrodes by successively grounding each link. J. Mech. Transm. Autom. **105**(3), 407–414 (1983)
10. Figliolini, G., Lanni, C., Tomassi, L.: Kinematic analysis of slider – crank mechanisms via the bresse and jerk's circles. In: Carcaterra, A., Paolone, A., Graziani, G. (eds.) AIMETA 2019. LNME, pp. 278–284. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-41057-5_23
11. Figliolini, G., Lanni, C., Cirelli, M., Pennestrì, E.: Kinematic properties of nth-order bresse circles intersections for a crank-driven rigid body. Mech. Mach. Theory **190**, 105445 (2023)
12. Figliolini, G., Lanni, C., Sorli, M.: Kinematic analysis and centrodes between rotating tool with reciprocating motion and workpiece, In: Niola, V., Gasparetto, A., Quaglia, G., Carbone, G. (eds.) Mechanisms and Machine Science, vol. 122 MMS, pp. 54–60 (2022). https://doi.org/10.1007/978-3-031-10776-4_7
13. Figliolini G., Lanni C., Tomassi L.: First and second order centrodes of slider-crank mechanisms by using instantaneous invariants. In: Altuzarra, O., Kecskeméthy, A. (eds.) 18th International Symposium on Advances in Robot Kinematics, ARK 2022, Springer Proc. in Advanced Robotics, Volume 24 SPAR, pp. 303–310, Bilbao, 26–30 June 2022 (2022). https://doi.org/10.1007/978-3-031-08140-8_33
14. Lanni, C., Figliolini, G., Tomassi, L.: Higher-order centrodes and Bresse's circles of slider-crank mechanisms. In: Proceedings of the ASME 2023 IDETC-CIE2023, 20–23 Aug 2023, Boston, Massachusetts, DETC2023-116627, vol. 8, v008t08a012 (2023)