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CORRIGENDUM

CORRIGENDUM: The inverse obstacle problem for nonlinear inclusions (2025 *Inverse Problems* **41** 055002)

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1. Introduction

1.1. The issues

In [2], devoted to solving the inverse obstacle problem for nonlinear materials via the Monotonicity Principle, two issues were found in the formulation of some hypotheses. Specifically, the second assumption in **(B2)**/(4.29), i.e.

$$0 < \underline{\gamma} \leqslant \gamma_{NL}(x,s) \leqslant \overline{\gamma}\left(\frac{s}{s_0}\right)^{q-2} \text{ for a.e. } x \in \overline{A} \text{ and } \forall s > 0 \tag{1.1}$$

is inconsistent for large s, when $1 < q < 2$. Indeed, for those values of q, the upper bound vanishes as $s \to +\infty$, thus contradicting the assumed lower bound. Similarly, the second assumption of **(B3)**/(4.43), i.e.

$$\underline{\gamma}\left(\frac{s}{s_0}\right)^{q-2} \leqslant \gamma_{NL}(x,s) \leqslant \overline{\gamma} < +\infty \text{for a.e. } x \in A,\ \forall s > 0,$$

is inconsistent for large s, when $q > 2$. Indeed, the lower bound diverges as $s \to +\infty$, which contradicts the existence of a finite upper bound $\overline{\gamma}$.

1.2. The corrections

Regarding the first issue, assumption **(B2)**/(4.29) for $1 < q < 2$ is inconsistent and must be removed. Consequently, assumption **(C2)** for $1 < q < 2$ is no longer required and must be removed.

Regarding the second issue, i.e. assumption **(B3)**/(4.43), it has to be modified as follows:

$$\gamma_{NL}(x,s) \leqslant \overline{\gamma} < +\infty \text{for a.e. } x \in A, \quad \forall s > 0,$$

and for $1 < q < 2$.

In order to get to the revised version of assumption **(B3)**, we need to revise assumption **(C3)** for the case $1 < q < 2$ and, consequently, we need to generalise the monotonicity results of paper [1]. This generalisation of the results of [1], in addition to solving the second issue, constitutes a stand-alone result and a kind of advancement of the underlying theory.

In addition to these required modifications, it is convenient to introduce a proper normalisation constant s_0 that has the same physical dimension as the underlying field $\mathbf{s} = -\nabla u$. This normalisation was already present in assumptions **(B2)** and **(B3)**, and it is now introduced for the assumptions **(C1)**, **(C2)**, and **(C3)**.

The corrected list of assumptions is given below.

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(B1) There exist two positive constants $\underline{\gamma} \leqslant \overline{\gamma}$ such that

$$0 < \underline{\gamma} \leqslant \gamma_{NL}(x,s) \leqslant \overline{\gamma} < +\infty \quad \text{for a.e. } x \in A \text{ and } \forall s > 0.$$

(C1) There exist two positive constants $\underline{\gamma}$ and s_0 such that

$$\left(\gamma_{NL}(x,s_2)\frac{\mathbf{s_2}}{s_0} - \gamma_{NL}(x,s_1)\frac{\mathbf{s_1}}{s_0}\right)\cdot\left(\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right) \geqslant \underline{\gamma}\left|\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right|^2,$$

for a.e. $x \in \overline{A}$ and for any $\mathbf{s}_1, \mathbf{s}_2 \in \mathbb{R}^n$.

(B2) For fixed $q \geqslant 2$, there exist three positive constants $\underline{\gamma} \leqslant \overline{\gamma}$ and s_0 such that

$$\underline{\gamma} \leqslant \gamma_{NL}(x,s) \leqslant \overline{\gamma}\left[1 + \left(\frac{s}{s_0}\right)^{q-2}\right],$$

for a.e. $x \in \overline{A}$ and $\forall s > 0$.

(C2) For fixed $q \geqslant 2$, there exist two positive constants $\underline{\gamma}$ and s_0 such that

$$\left(\gamma_{NL}(x,s_2)\frac{\mathbf{s_2}}{s_0} - \gamma_{NL}(x,s_1)\frac{\mathbf{s_1}}{s_0}\right)\cdot\left(\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right) \geqslant \underline{\gamma}\left|\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right|^q,$$

for a.e. $x \in \overline{A}$ and for any $\mathbf{s}_1, \mathbf{s}_2 \in \mathbb{R}^n$.

(B3) For fixed $1 < q < 2$, there exists a positive constant $\overline{\gamma}$ such that

$$\gamma_{NL}(x,s) \leqslant \overline{\gamma},$$

for a.e. $x \in \overline{A}$ and $\forall s > 0$.

(C3) For fixed $1 < q < 2$, there exist two positive constants $\underline{\gamma}$ and s_0 such that

$$\left(\gamma_{NL}(x,s_2)\frac{\mathbf{s_2}}{s_0} - \gamma_{NL}(x,s_1)\frac{\mathbf{s_1}}{s_0}\right)\cdot\left(\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right) \geqslant \underline{\gamma}\left(1 + \left|\frac{\mathbf{s_2}}{s_0}\right|^2 + \left|\frac{\mathbf{s_1}}{s_0}\right|^2\right)^{\frac{q-2}{2}}\left|\frac{\mathbf{s_2}}{s_0} - \frac{\mathbf{s_1}}{s_0}\right|^2,$$

for a.e. $x \in \overline{A}$ and for any $\mathbf{s}_1, \mathbf{s}_2 \in \mathbb{R}^n$.

Finally, we observe that assumption **(A2)**, i.e.

$$s \in [0, +\infty) \mapsto \gamma_{NL}(x,s)s \text{ is strictly increasing for a.e.} x \in A,$$

is redundant, since it is implied by each of the assumptions **(Cx)**. Indeed, for any $\nu \in \mathbb{R}^n$ with $|\nu| = 1$, by considering two vectors parallel to ν:

$$\mathbf{s}_1 = s_1\nu, \qquad \mathbf{s}_2 = s_2\nu,$$

with $s_2 > s_1 \geqslant 0$, and by substituting into **(C1)**, it results:

$$\big(\gamma_{NL}(x,s_2)s_2 - \gamma_{NL}(x,s_1)s_1\big)(s_2 - s_1) \geqslant \underline{\gamma}(s_2 - s_1)^2 > 0,$$

that concludes. Analogous conclusions follows by considering **(C2)** or **(C3)**.

1.3. Impact of the corrections on sections 4.3 and 4.4 in [2]

The revised assumptions impact two of the four converse results proven in [2]. Specifically, they impact the converse of the Monotonicity Principle when: (i) the anomaly is more conducting than the background and, moreover, its (nonlinear) electrical conductivity approaches $+\infty$ (section 4.3 of [2]), and (ii) the anomaly is less conducting than the background and its (nonlinear) electrical conductivity approaches 0 (section 4.4 of [2]). The other two cases, where the electrical conductivity of the anomaly is bounded from zero and from $+\infty$ (sections 4.1 and 4.2 of [2]), are unaffected by the present changes.

Specifically, the case $1 < q < 2$ in (B2)/(4.29) leads to an inconsistency, as clarified, and has been deleted, while the case $q \geqslant 2$ is consistent and does not require any modification. Similarly, the case $q \geqslant 2$ in (B3)/(4.43) leads to an inconsistency; however, it has been modified to account for $1 < q < 2$.

Summarising, theorem 4.8 holds only when $q \geqslant 2$ in its present form; while theorem 4.13 holds under the revised assumptions (B3)-(C3) introduced in this corrigendum, and only for $1 < q < 2$. No modifications in the proofs are needed.

Moreover, in remark 4.5 of [2], the statement concerning the case $q < 2$ and the final sentence about applications are incorrect and should be deleted. The corrected remark reads as follows:

Remark 4.5. Assumption (4.29) bounds the growth rate of the material property. The case $q \geqslant 2$ corresponds to unbounded material properties increasing with the amplitude of s ($\gamma \to +\infty$, for $s \to +\infty$).

Finally, figure 8 in [2] is inconsistent with the assumptions introduced in this corrigendum and it should be replaced by figure 2 of the present corrigendum.

2. Description of the generalization for [1]

In the present corrigendum we need to match the notation of [2] and that of [1]. Specifically, σ and $\mathbf{E}$ of [1] stand for γ and $\mathbf{s}$ of [2], respectively. The proofs in this corrigendum are given in the language used in [1].

The modifications proposed in this section concern assumptions **(P2)** and **(Q1)** of [1], since it has been found that the lower bounds are no longer needed.

In addition to these required modifications for **(P2)** and **(Q1)**, it is convenient to introduce a proper normalisation constant E_0 in assumptions **(P3)** and **(Q2)**, which has the same physical dimension as the underlying field $\mathbf{E}$.

In summary, the generalised version of the assumptions is as follows:

(P2) For fixed $1 < p < +\infty$, there exists two positive constants $\overline{\sigma}$ and E_0 such that:

$$\sigma_B(x,E) \leqslant \overline{\sigma}\left[1+\left(\frac{E}{E_0}\right)^{p-2}\right] \qquad \text{if } p \geqslant 2,$$
$$\sigma_B(x,E) \leqslant \overline{\sigma}\left(\frac{E}{E_0}\right)^{p-2} \qquad \text{if } 1 < p < 2,$$

for a.e. $x \in \overline{B}$ and $\forall E \geqslant 0$.

(Q1) For fixed $1 < q < +\infty$, there exists two positive constants $\overline{\sigma}$ and E_0 such that:

$$\sigma_A(x,E) \leqslant \overline{\sigma}\left[1+\left(\frac{E}{E_0}\right)^{q-2}\right] \qquad \text{if } q \geqslant 2,$$
$$\sigma_A(x,E) \leqslant \overline{\sigma}\left(\frac{E}{E_0}\right)^{q-2} \qquad \text{if } 1 < q < 2,$$

for a.e. $x \in A$ and $\forall E > 0$.

(P3) For fixed $1 < p < +\infty$, there exist two positive constants $\underline{\sigma}$ and E_0 such that:

$$\left(\sigma_B(x,E_2)\frac{\mathbf{E_2}}{E_0}-\sigma_B(x,E_1)\frac{\mathbf{E_1}}{E_0}\right)\cdot\left(\frac{\mathbf{E_2}}{E_0}-\frac{\mathbf{E_1}}{E_0}\right)$$
$$\geqslant \begin{cases} \underline{\sigma}\left|\dfrac{\mathbf{E_2}}{E_0}-\dfrac{\mathbf{E_1}}{E_0}\right|^p & \text{if } p \geqslant 2 \\ \underline{\sigma}\left(1+\left|\dfrac{\mathbf{E_2}}{E_0}\right|^2+\left|\dfrac{\mathbf{E_1}}{E_0}\right|^2\right)^{\frac{p-2}{2}}\left|\dfrac{\mathbf{E_2}}{E_0}-\dfrac{\mathbf{E_1}}{E_0}\right|^2 & \text{if } 1 < p < 2 \end{cases}$$

for a.e. $x \in B$, and for any $\mathbf{E}_1, \mathbf{E}_2 \in \mathbb{R}^n$.

(Q2) For fixed $1 < q < +\infty$, there exist two positive constants $\underline{\sigma}$ and E_0 such that:

$$\left(\sigma_A(x,E_2)\frac{\mathbf{E_2}}{E_0} - \sigma_A(x,E_1)\frac{\mathbf{E_1}}{E_0}\right)\cdot\left(\frac{\mathbf{E_2}}{E_0} - \frac{\mathbf{E_1}}{E_0}\right) \geqslant \begin{cases} \underline{\sigma}\left|\dfrac{\mathbf{E_2}}{E_0} - \dfrac{\mathbf{E_1}}{E_0}\right|^q & \text{if } q \geqslant 2 \\ \underline{\sigma}\left(1 + \left|\dfrac{\mathbf{E_2}}{E_0}\right|^2 + \left|\dfrac{\mathbf{E_1}}{E_0}\right|^2\right)^{\frac{q-2}{2}}\left|\dfrac{\mathbf{E_2}}{E_0} - \dfrac{\mathbf{E_1}}{E_0}\right|^2 & \text{if } 1 < q < 2 \end{cases}$$

for a.e. $x \in A$, for any $\mathbf{E}_1, \mathbf{E}_2 \in \mathbb{R}^n$.

The lower bounds in **(P2)** and **(Q1)** are no longer required, since weaker and proper lower bounds are provided by assumptions **(P3)** and **(Q2)**, respectively. Indeed, for $\mathbf{E}_1 = \mathbf{0}$ and $\mathbf{E}_2 = \mathbf{E}$ in **(P3)**, it follows that

$$\sigma_B(x,E) \geqslant \begin{cases} \underline{\sigma}\left(\dfrac{E}{E_0}\right)^{p-2}, & \text{if } p \geqslant 2 \\ \underline{\sigma}\left[1 + \left(\dfrac{E}{E_0}\right)^2\right]^{\frac{p-2}{2}}, & \text{if } 1 < p < 2, \end{cases} \tag{2.1}$$

for a.e. $x \in B$, for any $E > 0$. Similarly, **(Q2)** gives the following lower bound

$$\sigma_A(x,E) \geqslant \begin{cases} \underline{\sigma}\left(\dfrac{E}{E_0}\right)^{q-2}, & \text{if } q \geqslant 2 \\ \underline{\sigma}\left[1 + \left(\dfrac{E}{E_0}\right)^2\right]^{\frac{q-2}{2}}, & \text{if } 1 < q < 2, \end{cases} \tag{2.2}$$

for a.e. $x \in A$, for any $E > 0$.

It is worth noting that the lower bounds given by (2.1) and (2.2), for $p \geqslant 2$ and $q \geqslant 2$ are equivalent to the lower bounds for assumptions **(P2)** and **(Q1)**, as originally stated in [1].

The situation is completely different for $1 < p < 2$ and $1 < q < 2$. Indeed, the proposed weakening of the assumptions is not only a technical matter, but it also allows us to expand the class of nonlinear materials (see figure 1) that can be treated within the framework of the Monotonicity Principle. As shown in figure 1 left), under the original assumptions of [1], the Monotonicity Principle is applied to electrical conductivities that (i) must be unbounded as $E \to 0^+$ and (ii) vanish as $E \to +\infty$. On the contrary, under the new proposed assumptions, the electrical conductivity can also be bounded as $E \to 0^+$, as shown in figure 1 right).

Consequently, figure 8 in [2] should be replaced by figure 2 of the present corrigendum.

Therefore, to show that the inequalities on the left-hand side of **(P2)** and **(Q1)** can be removed, a key point is to determine whether this weakening of the assumptions still suffices to guarantee the monotonicity result established in [1]. Specifically, the left-hand sides of **(P2)** and **(Q1)** enter these two entry points:

(1) The coercivity of the Dirichlet Energy density $Q(\cdot,\cdot)$, as proved in theorem 2.7 of [1].

(2) The first inequality of chain (3.5) of [1] that appears in the proof of claim (i) of lemma 3.1 of [1], and the analogous inequality appearing in the proof of claim (ii) of the same lemma 3.1.

2.1. Treatment of the first entry point

Here, the estimate (2.2) of [1] is re-evaluated from below by using (2.1) and (2.2) of this corrigendum.

We recall that the Dirichlet Energy density is defined as

$$Q(x,E) := \int_0^E \sigma(x,\xi)\,\xi\,\mathrm{d}\xi,$$

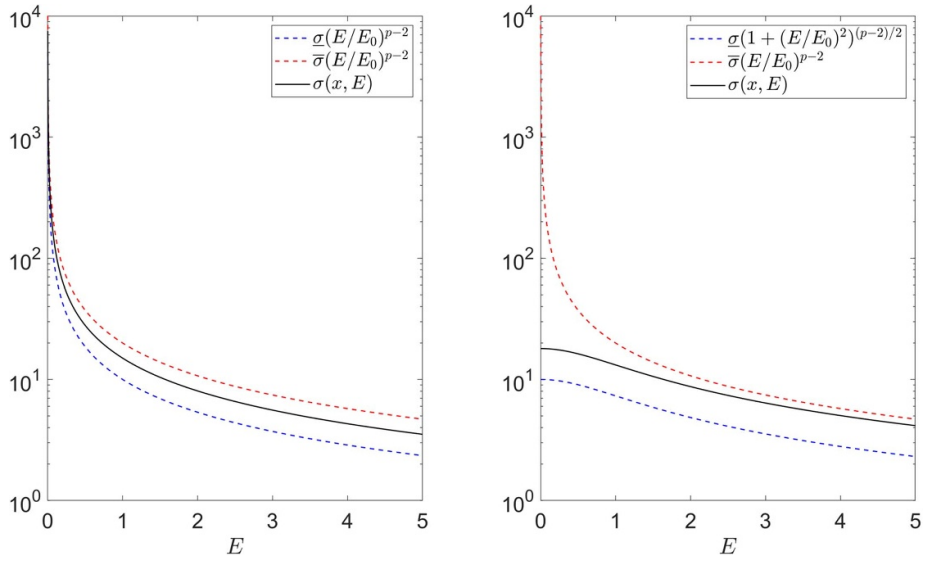


Figure 1. Comparison of conductivity hypotheses for $1 < p < 2$. Left: electrical conductivity fulfilling the hypothesis in [1]; the upper and lower bounds, and, hence, the electrical conductivity, are singular in $E = 0$. Right: electrical conductivity fulfilling the new hypothesis; the upper bound only is singular in $E = 0$. The lower bound is no longer approaching $+\infty$ and, therefore, the electrical conductivity does not need to approach $+\infty$ and may approach a finite limit. The figure is obtained for $p = 1.1$, $\overline{\sigma} = 20$ S m^{-1}, $\underline{\sigma} = 10$ S m^{-1}, and $E_0 = 1$ Vm$^{>-1}$.

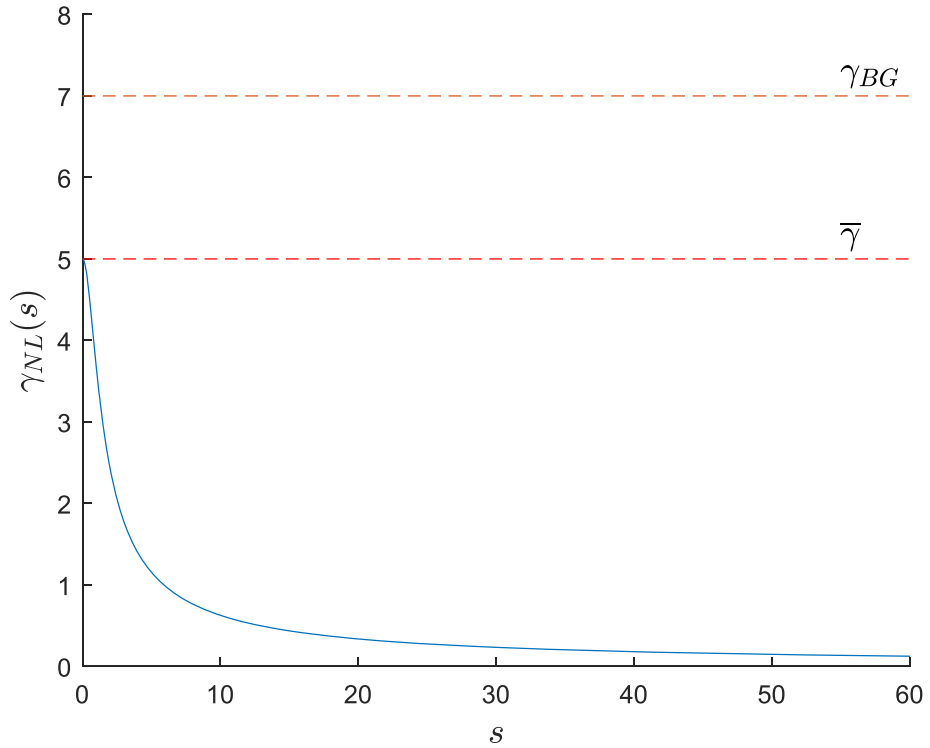


Figure 2. Revised figure 8 in [2].

and its lower bound can be found by exploiting the new lower bounds given by (2.1) and (2.2). Specifically, a straightforward computation gives

$$Q(x,E) \geqslant \begin{cases} \dfrac{\underline{\sigma}}{pE_0^{p-2}}\,E^p, & \text{if } p \geqslant 2, \\ \dfrac{\underline{\sigma}E_0^2}{p}\left[\left(1+\left(\dfrac{E}{E_0}\right)^2\right)^{\frac{p}{2}}-1\right], & \text{if } 1<p<2, \end{cases} \tag{2.3}$$

for a.e. $x \in B$ and for any $E > 0$. Finally, a further lower bound valid for any $p > 1$ is given by

$$Q(x,E) \geqslant \frac{\underline{\sigma} E_0^2}{p}\left[\left(\frac{E}{E_0}\right)^p - 1\right], \quad \text{if } p > 1, \tag{2.4}$$

for a.e. $x \in B$ and for any $E > 0$.

A similar estimate holds in the region A with the exponent q. This provides an explicit uniform lower bound for $Q(x,E)$, that is

$$Q(x,E) \geqslant \frac{\underline{\sigma} E_0^2}{\max\{p,q\}}\left[\min\left\{\left(\frac{E}{E_0}\right)^p, \left(\frac{E}{E_0}\right)^q\right\} - 1\right], \tag{2.5}$$

for a.e $x \in \Omega$ and for any $E > 0$.

2.2. Treatment of the second entry point

Here, the leftmost inequality in (3.5) of [1] is re-evaluated by using (2.4) of this corrigendum.

Specifically, by using the first inequality of (2.4)

$$\begin{aligned}\int_\Omega Q(x, |\nabla u^f(x)|)\,\mathrm{d}x &\geqslant \underline{c}\int_\Omega (|\nabla u^f(x)|^p - E_0^p)\,\mathrm{d}x \\ &= \underline{c}\int_\Omega |\nabla u^f(x)|^p\,\mathrm{d}x - \underline{c}E_0^p|\Omega|.\end{aligned}$$

where

$$\underline{c} = \frac{\underline{\sigma}}{pE_0^{p-2}}$$

is a proper constant introduced in [1].

Therefore, the upper bound to the p-norm of ∇u^f after the revised form of (3.6) of [1], becomes

$$||\nabla u^f||_p^p \leqslant C_1(C_2 + (C_2+1)K_p^p||f|||_{X^p(\partial\Omega)}^p) + E_0^p|\Omega|.$$

This gives the claim with $C = \max\{C_1C_2 + E_0^p|\Omega|, C_1(C_2+1)K_p^p\}$.

A similar approach must be followed for the analogous inequality appearing in the proof of claim (ii) of the same lemma 3.1, by exploiting (2.4) of this corrigendum with p replaced by q.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors. No new data were created or analysed in this study.

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References

[1] Esposito A C, Faella L, Mottola V, Piscitelli G, Prakash R and Tamburrino A 2024 Piecewise nonlinear materials and monotonicity principle *Inverse Problems* **40** 1–31
[2] Mottola V, Esposito A C, Faella L, Piscitelli G, Prakash R and Tamburrino A 2025 The inverse obstacle problem for nonlinear inclusions *Inverse Problems* **41** 055002