

Book of the Short Papers

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Preface

This book includes the contributions presented at the Intermediate Meeting of the Italian Statistical Society (SIS) "SIS 2023 - Statistical Learning, Sustanaibility and Impact Evolution" held in Ancona at the Università Politecnica delle Marche, from June 21th to 23th of 2023.

The new challenges of digitalization, innovation and sustainability are showing the crucial role of data-driven approaches in supporting decision-making processes. Methodologies resulting from the integration of different know-how seem to be a reliable way to deal with the increasing need to measure the impact of the policies and to forecast scenarios. This meeting welcomed any attempt to face new challenges.

The conference registered more than 250 presentations, including 3 keynote speakers in 3 plenary sessions and 72 presentations in 24 invited sessions, all dealing with specific themes in methodological and/or applied statistics and demography. Furthermore, more than 180 contributions, with one or more authors, have been spontaneously submitted to the Program Committee and arranged in 30 contributed sessions.

The numerous participation of researchers in the conference shows how the challenges of sustainability, in its broadest sense, are of interest to both methodological and applied statistics.

With the publication of this book, we wish to offer to all members of the Italian Statistical Society, all international academics, researchers, Ph.D. students, and all interested practitioners, a good snapshot of the on-going research in the statistical and demographic fields.

We aim to provide all members of the Italian Statistical Society - as well as international academics, researchers, Ph.D. students, and interested practitioners - with a comprehensive overview of the ongoing research in the fields of statistics and demography.

We extend our heartfelt gratitude to all the contributors for submitting their works to the conference and to the researchers for their outstanding job in serving as referees and discussants with precision and timeliness.

A special appreciation goes to the Scientific and Organizational Committees for their tremendous efforts in managing all the organizational aspects, as well as to the Università Politecnica delle Marche and the Department of Economic and Social Science for making this event possible.

Finally, we wish to express our gratitude to the publisher Pearson Italia for all the support received.

Enhancing Principal Components by a Linear Predictor: an Application to Well-Being Italian Data

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a

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Abstract

We consider the case of a multivariate random vector that obeys a linear mixed model when the vector itself lies in a lower dimensional subspace. This situation suggests that this subspace can be modeled by the probabilistic (random-effects) principal components. By reason of this, the random vector follows at the same time two different models. We employ a linear predictor adjusted by the residual part of the probabilistic principal components that results not explained by the linear model. The new predictor can be considered as the vector of scores that comes from that principal components, enhanced by the linear mixed model. The application to the official Italian well-being data shows some features of the method.

Keywords: multivariate random vector, principal components, linear mixed model, well-being

1. Introduction

Principal component analysis (PCA) is recognized as one of the most employed methods to reduce dimensionality, by means of the projection of a set of variables in a subspace of them. By summarizing and allowing to visualize data, and, at the same time, minimizing the loss of information in the lower dimensional space, in many cases principal components (PCs) lead to a better assessment of the bundled statistical information, seized by the original variables [\[2\]](#page-26-0). Because PCs are linear combinations, the interpretation of the scores by these new "data-dependent" variables is hard to give some time. In particular situations, the contribution improve understanding of some case studies may be poor and may lead to misguided or unclear findings. Furthermore, in the great majority of cases, when the interpretation stills on loadings that exceed a threshold, the linking of the variables hardly fails to provide some explanation or a bit of insight. Because of using of the common practice of ignoring the PCs affected by lower loadings, the focus shifts to the first PCs, which arise from the most correlated variables in the original set. The issue of resting on the main linearly-dependent variables is particularly relevant and becomes crucial in several instances. We may come across the trade-off between considering the retained PCs as highlighting latent phenomena, or in reproducing similar information unnecessarily. One of the recurrent ways of approaching redundancy and, in general, the recursive informative content of multivariate sample data, is given by considering a common subset of covariates that the population obeys. Two main cases in the literature are deemed representative of the joint dependence on a multivariate vector, the PCs "with covariates" or Partial PCA, and the redundancy analysis. Given a subspace spanned by the sample vectors of predictor variables, both of them rely on the common baseline of splitting the sample variance as the sum of the variance "explained" by a multivariate linear regression model, and the variance due to the regression residuals. Although these last represent a very useful tool in some cases, the deployment of linear models to explain part of the sample variability has had a remarkable development in the last years. One of these studies brings into play the role of prediction by linear statistical models.

Tipping and Bishop [\[7\]](#page-26-1) had already introduced the notion of prediction for PCs. They called "Probabilistic PCA" (PPCA) the model behind the PCA, which parameters are estimated with the Expectation - Maximization algorithm. The "noisy" PC model (nPC), proposed by Ulfarsson and Solo [\[8\]](#page-26-2) has a quite similar formulation with respect to the PPC model, providing - in a similar way - the nPC prediction after giving the model estimates. Instead of a "fixed effects PCs", as the traditional linear regression PCA model, the PPC (or nPC) are random variables. This condition suggests, on the one hand, the Bayesian approach to handle the estimates for the PPC linear model and, on the other hand, to predict PCs under its meaning within random linear models theory [\[4\]](#page-26-3). The Bayesian approach to the estimation requires an expectation of some model parameters that are random, conditional to the observed data. Given normality of the error $\varepsilon \sim N(0, \sigma^2 I)$, for a linear model $\tau = B\lambda + \varepsilon$ - in case of the vector λ random - the likelihood is based on the conditional distribution $\lambda | \tau \sim N[E(\lambda | \tau), var(\lambda | \tau)].$

Moreover, it is known [\[6\]](#page-26-4) that $E(\lambda|\tau) = \lambda$ is the "Best Prediction" (BP) estimate, with $var(\lambda \lambda$) = E_{τ} [var($\lambda|\tau$)]. This is somewhat different from the standard linear regression model prediction by $E(\tau|\lambda)$. Therefore, given a linear mixed model (LMM) [\[1\]](#page-26-5) for τ , with $E(\tau|\lambda) = \lambda$, the model parameters are the realizations of random variables. The BP of a linear combination of the LMM fixed and random effects (i.e. linear in τ , with $E[E(\tau|\lambda)] = 0$) gives the "Best Linear Unbiased Prediction" (BLUP) estimates [\[5\]](#page-26-6). LMM's are particularly suitable for modeling with covariates (fixed and random) and for specifying model covariance structures [\[1\]](#page-26-5). They allow researchers to take in account special data, such as hierarchical, time-dependent, correlated, covariance-patterned models. Thus, given the BP estimates of the nPC λ , $\lambda = E(\lambda|\tau)$, the vector $\tilde{\tau} = B\lambda$ represents the BP of the *p*-variate vector.

In the present paper, we introduce a multivariate LMM that considers the dependent random vector effectively represented by the subspace of the PPCs. The new predictor combines the linear model and the PPC's, carrying simultaneously the Best Linear Predictor, and the contribution given by the PPCs not "explained" by the linear predictor itself. An application to the official Well-being Italian indicators shows some of the features of the method.

2. Theory

In the sequel we report the following symbols, giving the model specification.

- *n* = the number of subjects in the LMM model $(i = 1, ..., n)$;
- $N = \sum n_i$ = total sampling units considered;
- $p =$ the number of the response dependent variables;
- \bullet *l* = the number of the linear model covariates:
- $j = 1, ..., n_i$ the within-subject (groups) units;
- $s =$ the dimension of the effective PC subspace.

Consider Θ as the *N* \times *p* sample matrix of the *p*-variate $p \times 1$ random vector θ , with *N* as the total number of the units given by the sample. Moreover, consider that the vector θ obeys the linear model:

$$
\theta = \beta' x + u',\tag{1}
$$

where *x* is the $l \times 1$ vector of covariates, β is the $l \times p$ matrix of the regression effects, *u* is the vector of the *p*-variate random effect, with $u \sim N(0, \Sigma_u)$, $\Sigma_u = cov(u)$. Furthermore, we consider at the same time that the multivariate random vector θ obeys the following linear model:

$$
\theta = Ab + \epsilon,\tag{2}
$$

in which *A* is $p \times s$ a loading matrix of eigenvectors, *b* is the random vector of PPCs, and ϵ is a vector of isotropic error, with $\theta \sim N(\mu, A\Psi A' + \sigma_{\epsilon}^2 I)$, $b \sim N(0, \Psi)$, $\Psi = diag(\psi_1, \dots, \psi_s)$, $s < p$, and $\epsilon \sim N(0, \sigma_{\epsilon}^2 I)$. When a sample of *N* observations is given, an $N \times p$ matrix *Y* of observations from the random vector θ is simply modeled as $Y = \Theta + E$, with the "sampling error" $Np \times Np$ covariance matrix

 $cov(vec(E)) = (\Sigma_e) \otimes I_N \otimes \Sigma$ is the Kronecker product), $e \sim N(0, \Sigma_e)$, $\Sigma_e = var(e)$. Thus, models [\(1\)](#page-23-0) and [\(2\)](#page-23-1) are rewritten as $Y = \Theta + E = X\beta + ZU + E$, with the [\(1\)](#page-23-0) that becomes $\theta = \beta'x + u' + e'$, and $Y = \Theta + E = BA' + \Xi + E = BA' + \Gamma$, with the [\(2\)](#page-23-1) is $\theta = Ab + \epsilon + e = Ab + \gamma$, respectively. The model errors u , ϵ , and e , are mutually independent. The matrix *Z* represents the $N \times n$ design matrix of random effects and *E* is the *N* \times *p* matrix of the residual errors of the multivariate LMM, *B* is the *N* \times *s* matrix of the PPCs that lie in the *s*-dimensional subspace, Ξ is the $N \times p$ matrix of the isotropic errors of the model (2) . The models (1) and (2) have the following conditional expectations and variances:

$$
E(\theta|y) = \tilde{\theta}_y = y - E(e|y) = y - cov(e, y)var(y)^{-1}y = y - var(e)Py,
$$

\n
$$
var(\theta|y) = var(\theta) - cov(\theta, y)var(y)^{-1}cov(y, \theta)
$$

\n
$$
= var(e) - var(e)Pvar(e),
$$

for the model in [\(1\)](#page-23-0), where $P = \sum_{y}^{-1} (I - P_X)$, $\Sigma_y = var(y)$, and P_X is the projection matrix. For the model in [\(2\)](#page-23-1):

$$
E(b|\theta) = E(b) + cov(b, \theta)var(\theta)^{-1}(\theta - \mu)
$$

= cov(b, \mu + Ab + \epsilon)C^{-1}(\theta - \mu) = \Psi A'C^{-1}(\theta - \mu),
var(b|\theta) = var(b) - cov(b, \theta)var(\theta)^{-1}[cov(b, \theta)]'
= \Psi - \Psi A'C^{-1}A\Psi
C = A\Psi A' + \sigma_{\epsilon}^2 I.

Based on some results on linear projections, i.e., given the random variable *y*, and the $1 \times j$, $1 \times k$ random vectors *x*, *z*, with positive definite covariance matrix of $(y, x, z)'$, then for the linear projection $L(y|x, z)$:

$$
L(y|x, z) = L(y|x) + [z - L(z|x)]\gamma,
$$

where $\gamma = var(z|x)^{-1}[cov(y, z|x)]'$, we get the following:

Proposition 1. *Given the model* [\(2\)](#page-23-1) *for the p-dimensional random vector* θ *, with* $b = \overline{F}'(\theta - \epsilon)$ *, and under the models in* [\(1\)](#page-23-0) *and* [\(2\)](#page-23-1)*, the multivariate Best Predictor based on* (y, b) *,* $E(\theta | y, b)$ *, is:*

$$
E(\theta|y,b) = \widetilde{\theta}_{y,b} = E(\theta|y) + cov(\theta,b|y)var(b|y)^{-1} \left\{ \widetilde{b} - E(b|y) \right\},\tag{3}
$$

with $b = E(b|\theta)$, \overline{F} the sN $\times pN$ matrix $(\overline{A'A})^{-1}\overline{A}$, and \overline{A} is the $pN \times sN$ matrix $A \otimes I_N$. Then, $var(\theta | y, b) = var(\theta | y) - cov(\theta, b | y) var(b | y)^{-1} [cov(\theta, b | y)]'.$

The "hybrid" predictor $\theta_{y,b}$ in [\(3\)](#page-24-0) gives the Best Linear Unbiased Predictor $E(\theta|y)$, "embedding" the PPCs through an adjoint component. The last is due to knowing that the random vector θ lies in the *s*-dimensional subspace of the PPCs. In particular, the difference $\tilde{b} - E(b|y)$ gives the multivariate vector of the PPCs "not explained" by the estimation of the linear model $E(\theta|y)$. The matrix $var(\theta|y, b)$ has rank *s*, and, consequently, there are $(p - s)$ linear combinations of θ for which their respective variances do not depend on the PPCs.

Proposition 2. *Given the p-dimensional random vector* θ *, under the models in* [\(1\)](#page-23-0) *and* [\(2\)](#page-23-1)*, and the Best Predictor* $E(\theta | y, b)$ *in* [\(3\)](#page-24-0)*, we get:*

$$
\overline{F}E(\theta|y,b) = \overline{F}\widetilde{\theta}_{y,b} = \widetilde{b}^*
$$

=
$$
\overline{F}E(\theta|y) + \overline{F}cov(\theta,b|y)var(b|y)^{-1} \left\{ \widetilde{b} - E(b|y) \right\},
$$
 (4)

where \widetilde{b}^* *is the s-dimensional vector of the PPCs "enhanced" (ePCs) by the linear predictor* $E(\theta|y)$. $As \alpha$ particular case, when $\sigma_{\epsilon}^2 \longrightarrow 0$, $var(\epsilon) \longrightarrow 0$, $var(\gamma) \longrightarrow var(e)$, and $\tilde{b} \longrightarrow \bar{b}$. Therefore, $\overline{F}\theta_{y,b} = b^* \longrightarrow b$ with *b* the sample PCs $b = A'\theta$.

The ePCs \tilde{b}^* are then the PPCs "adjusted" by the Linear BP $E(\theta|y)$, and \tilde{b}^* is then the vector of the ePC scores. Note that the ePC scores give a non-orthogonal matrix. Given $\sigma_{\epsilon}^2 = 0$, the vector θ in the model [\(2\)](#page-23-1) lies in the *s*-dimensional subspace of the sample PCs \bar{b} . In fact, in this case $\tilde{b} \equiv \bar{b}$, $cov(\theta, b|y) = var(\theta|y)\overline{F}', var(b|y) = \overline{F}var(\theta|y)\overline{F}', E(b|y) = \overline{F}E(\theta|y)$, and then $\overline{F}E(\theta|y, b) = \overline{b}$.

3. Application

In accordance with the recent law reforms in Italy, the Equitable and Sustainable Well-being indicators (in Italian, BES) [\[3\]](#page-26-7) - annually provided by the Italian Statistical Institute (ISTAT) - are designed to define the economic policies which largely act on some fundamental aspects of the quality of life. In order to highlight the result of the proposed method we use 12 BES indicators relating to the years 2013-2016, collected at NUTS-2 (Nomenclature of Territorial Units for Statistics 2 level). The variables employed in the application study are in Table [1.](#page-27-0) We use the per capita adjusted disposable income variable (its logarithm, as is usually done in economics studies) - indicated with BE1 - as a unique covariate in the LMM model, while the remaining 11 variables are dependent variables (Table [1](#page-27-0) reports the description and acronyms used for the variables). The application uses the Restricted Maximum Likelihood estimation, a Sas/IML code, and a sequence of Sas-HPMixed procedures. Table [2](#page-27-1) shows the slope parameter estimates from the multivariate regression, with their significance level. Table [3](#page-27-2) reports the MANOVA multivariate test statistics, based on the characteristic roots. These are the eigenvalues of the product of the sum-of-squares matrix of the regression model and the sum-of-squares matrix of the regression error. The null hypothesis for each of these tests is the same: the independent variable (LBE1) has no effect on any of the dependent variables. The four tests are all significant.

Figure [1](#page-26-8) shows the application of Proposition [1,](#page-24-1) where all the measures are plotted in the space of the sample PCs. This plot reports simultaneously the factorial coordinates of the original variables, of the linear predictor $E(\theta|y)$, and of the hybrid predictor $\theta_{y,b} = E(\theta|y, b)$. The dependent criterion variables in the application can be split into two main groups, starting from both an analysis of the plot and the correlation matrix between the sample PCs, the LMM predicted values, and the hybrid predictor values. Moreover, we have eight dependent variables for which there is accordance in terms of their mutual correlation inside the original variables, as well as the LMM predicted and the hybrid predictor. This means that the hybrid predictor $\theta_{y,b}$ does not change significantly the mutual correlations, substantially because the component of the PPCs not explained by the linear predictor $E(\theta|y)$ is relatively small. For the remaining three variables - with the acronyms REL4, Q2, and BS3 - the correlation changes: in some cases it changes sign, going from positive correlation values by the sampling and predicted values, to negative correlation values between the predictor $\theta_{y,b}$, and vice versa. Therefore the predictor [\(3\)](#page-24-0) highlights the major influence of the component of the PPCs not explained by the linear predictor $E(\theta|y)$. The latter is in accordance with the sample PCs in the mutual correlations between these three criterion variables. Since the classical predictor matches the mutual correlation inside the original variables meaning that these mutual relations in the sample are due to the disposable income (BE1), the covariate in the mixed model regression - then the different mutual correlation values of the hybrid predictor $\theta_{y,b}$ can be interpreted as relationships not captured by the model. For instance, the correlation between $Q2_{E(\theta|y)}$ and $INNI_{E(\theta|y)}$ is -0.82, and becomes +0.30 between $Q2_{\widetilde{\theta}_y,b}$ and $INNI_{\widetilde{\theta}_y,b}$, meaning that conditionally to the model - thus looking at the correlation between the attendance of childhood services (Q2) and the percentage of R&D (INN1) in the space orthogonal to per capita income - the correlation is positive. It could be interpreted as saying that the Regions with a greater investment in R&D have a higher benefit of childhood services. In our opinion, this highlights how the "hybrid" predictor is able to grasp the relationship that actually exists between investment in research and development and the importance given to training starting from the earliest years of life, regardless of per capita income.

4. Discussion

The introduced predictor [\(3\)](#page-24-0) can be viewed from two different perspectives. An "adjusted" linear predictor by the PPCs, and, by relation [\(4\)](#page-24-2), as the enhanced PCs (ePCs) that modify the probabilistic PCs (PPCs) to accommodate the mixed model regression predicted values. The present work considers the probabilistic principal components like a "constraint" model, that links together the components of the multivariate random vector in a lower dimensional subspace.

While the estimation of the PPC model requests a quite simple procedure, one of the causes of concern in the estimation of the parameters of a multivariate mixed model is the number of covariance

Figure 1: Plot of the application of the model [\(1\)](#page-23-0) in the space of the sample PCs. There are represented the factorial coordinates of the original variables, of the linear predictor $E(\theta|y)$, and of the hybrid predictor $\theta_{y,b} = E(\theta | y, b)$.

parameters, which may be too high for a speed software computation. Like the application presented, we suggest estimating the model covariance parameters under a uniform correlation structure among the multivariate components of the random effects. This structure is equivalent to the compound-symmetry covariance structure, with a better numerical property in terms of optimization. Indeed, some studies highlight that using uniform correlation matrices reduces the estimation noise. The model covariance matrix of random effects is then a generalized uniform correlation matrix, and works with two parameters. The "hybrid" multivariate linear predictor (3) , by adjusting its standard formulation through the sample parameter vector scores in a convenient subspace, is designed to accommodate not only PPCs, but also factor models based on a random structure. In the present work, the components of the multivariate parameter among the subjects are linked by a principal components model. In order to overcome convergence problems, the method introduced can be extended to include multidimensional information from the data, through a reduced number of dependent variables in the linear model.

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Variables	Description		
S ₈	Age-standardised mortality rate for dementia and nervous		
	system diseases		
IF3	People having completed tertiary education (30-34 years old)		
L12	Share of employed persons who feel satisfied with their work		
REL ₄	Social participation		
POL ₅	Trust in other institutions like the police and the fire brigade		
SIC ₁	Homicide rate		
BS3	Positive judgment for future perspectives		
PATR9	Presence of Historic Parks/Gardens and other Urban Parks		
	recognised of significant public interest		
AMB9	Satisfaction for the environment - air, water, noise		
INN1	Percentage of R&D expenditure on GDP		
Q ₂	Children who benefited of early childhood services		
BE1	Per capita adjusted disposable income		
LBE1	Logarithm of Per capita adjusted disposable income		

Table $1:$ Description of the variables used for the application

Table 2: The slope parameters by the multivariate regression with the LBE1 covariate

Dependent variable	Slope parameter (LBE1)	STD Error		Pr > t
AMB9	0.9802	0.3255	3.01	0.0035
BS3	0.9330	0.0891	10.47	0.0001
IF3	-0.3166	0.1673	-1.89	0.0621
INN1	-0.0433	0.0170	-2.54	0.0130
L12	0.0016	0.0107	0.15	0.8786
PATR9	0.0975	0.0756	1.29	0.2007
POL ₅	-0.0036	0.0085	-0.42	0.6775
Q ₂	0.2031	0.1762	1.15	0.2526
REL ₄	0.5602	0.1690	3.31	0.0014
S ₈	-0.0506	0.0293	-1.73	0.0879
SIC ₁	-0.0072	0.0150	-0.48	0.6314

Table 3: MANOVA Test Criteria and F Approximations for the Hypothesis of No Overall LBE1 Effect

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks Lambda	0.0566	102.98		68	< 0.001
Pillai's Trace	0.9434	102.98		68	< 0.001
Hotelling-Lawley Trace	16.6590	102.98		68	< 0.001
Roy's Largest Root	16.6590	102.98		68	< 0.001