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HIGHER ORDER KINEMATIC ANALYSIS OF LONG-DWELL MECHANISMS

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ABSTRACT

This paper deals with the higher order kinematic analysis of a long-dwell mechanism, which is synthesized by applying the dead-points superposition method. In particular, a crank-rocker four-bar linkage, a centered slider-crank mechanism, an orthogonal Cardan mechanism and an offset slider-rocker mechanism, are connected in series to give a crank-driven 10-bar long dwell mechanism, which kinematic analysis is formulated for the first time, up to the sixth-order and thus, including velocity, acceleration, jerk, jounce or snap, crackle and pop. The proposed formulation was validated by significant graphical and numerical results, which show the long dwell-time of the output rocker link for a constant angular velocity of the driving crank. Other multi-link mechanisms can be assembled in different way by using the dead-points superposition method and thus, obtaining a long-dwell mechanism.

Keywords: Higher order kinematic analysis, long dwell mechanisms, dead-points superposition method.

1. INTRODUCTION

The higher-order time derivatives of the position vector are of great importance in practical engineering, as for designing high-speed automatic machines. The jerk, the time rate of change of the acceleration, has some typical applications in cams and Geneva mechanisms design and analysis [1-4].

The indexing mechanisms are used to generate an intermittent motion, but in many applications linkages can be used for the same purpose. In fact, dwell mechanisms are often used in automatic machines to generate intermittent motions with a suitable holding position of the output member, but from the point of view of the design of the desired movement, the kinematic synthesis is more complicated. However, the kinematic synthesis of these mechanisms is traditionally formulated through two different approaches. The first is the one that refers to some important geometric loci of interest, such as the inflection circle, the cubic of stationary curvature and the Ball point [5-9], in order to obtain a dwell configuration that is related to two, three or four infinitesimal displacements, respectively.

The Burmester theory has also been largely used through years with the aim of designing dwell and long-dwell mechanisms, as reported in [10-15].

The second approach is the one that refers to the dead-point superposition method, obtained by assembling in series a train of linkages at dead-point position [16]. For example, a four-bar linkage, a slider-crank/rocker and double-slider mechanisms can be used for this purpose, as proposed in [17] and [18].

The analysis of the kinematic properties of *N*-bar long-dwell mechanisms, which are synthesized by applying the dead-points superposition method, is the main goal of this research activity. In particular, several four-bar linkages of any type, but able to give dead-points configurations, are assembled in series and according to their output rigid body motion, in order to obtain a suitable *N*-bar long-dwell mechanism, with *N* an even number.

This analysis is also addressed to design *N*-bar long-dwell mechanisms with assigned dwell-time and kinematic characteristics, in terms of number and type of the assembled four-bar mechanism with dead point configurations, which can be of 4R, 3RP, 2R2P and RPRP types.

In this paper, the higher order kinematic analysis of a 10-bar long-dwell mechanism, which is obtained by assembling in series four 4-bar linkages, *i.e*. a crank-rocker four-bar linkage, a centered slider-crank mechanism, an orthogonal Cardan mechanism or elliptic trammel and an offset slider-rocker mechanism, is formulated up to the sixth-order. In particular, the 10-bar long-dwell mechanism is driven by the crank of the four-bar linkage, in order to transmit the motion to the output rocker link, which shows a very long dwell time.

2. HIGHER ORDER KINEMATIC ANALYSIS

Referring to the sketch of Fig. 1, the type synthesis of the proposed 10-bar long dwell mechanism has been carried out by assembling in series four 4-bar linkages and in particular, the crank-rocker four-bar mechanism A_0ABB_0, the slider-crank mechanism B_0BC, the orthogonal Cardan mechanism CD, the offset slider-rocker mechanism DEE_0. Of course, this approach can be extended to other assembly modes and combinations of four-bar linkages of 4R, 3RP, 2R2P and RPRP types by applying the dead-points superposition method.

The planar positions of the fixed revolute joints A_0, B_0 and E_0, and the axes of the two prismatic pairs of pistons 6 and 8, are given by the angle θ_1 and the distance A_0B_0 of length r_1, along with the distances l and e of E_0 by the straight paths of points C and D, respectively. The distance of B_0 by the prismatic pair axis with piston 8 is h and the link lengths are given by r_i for i = 1 to 5, and also for i = 7, 9 and 10.

Moreover, the driving crank 2 is supposed to be moved with a constant angular velocity and θ_2, ω_2, α_2, φ_2 and ψ_2 are the crank angle and the angular velocity, acceleration, jerk, jounce or snap, crackle and pop, respectively.

The kinematic analysis of the proposed 10-bar long-dwell mechanism of Fig. 1 is formulated through four vector-loops, which are shown in Fig. 2 for each of the assembled linkages.

Thus, the following closed-loop equations can be written

$$\mathbf{r}_2 + \mathbf{r}_3 = \mathbf{r}_1 + \mathbf{r}_4 \quad (1)$$

$$\mathbf{r}_4 + \mathbf{r}_5 = \mathbf{r}_{B_0C} \quad (2)$$

$$\mathbf{r}_{CO'} + \mathbf{r}_{O'D} = \mathbf{r}_7 \quad (3)$$

$$\mathbf{e} + \mathbf{r}_{FD} + \mathbf{r}_9 = \mathbf{r}_{10} \quad (4)$$

where vectors $\mathbf{r}_i$ are expressed by

$$\mathbf{r}_i = \left[r_i \cos\theta_i, \quad r_i \sin\theta_i\right]^T \quad \text{for } i = 1 \text{ to } 5 \quad (5)$$

and for i = 7, 9 and 10, where T indicates the transpose vector, while vectors $\mathbf{r}_{B_0C}$, $\mathbf{r}_{CO'}$, $\mathbf{r}_{O'D}$, and $\mathbf{r}_{FD}$ are given by

$$\mathbf{r}_{B_0C} = \left[0, \quad r_1 \sin\theta_1 + y_C\right]^T \quad (6)$$

$$\mathbf{r}_{CO'} = \left[0, \quad h - r_1 \sin\theta_1 - y_C\right]^T \quad (7)$$

$$\mathbf{r}_{O'D} = \left[r_7 \cos\theta_7, \quad 0\right]^T \quad (8)$$

$$\mathbf{e} = \left[0, \quad e\right]^T \quad (9)$$

$$\mathbf{r}_{FD} = \left[x_{E0} - x_D, \quad 0\right]^T \quad (10)$$

where $x_{E0} = l + r_1 \cos\vartheta_1$ in Eq. (10).

Therefore, the position vectors $\mathbf{r}_B$, $\mathbf{r}_C$, $\mathbf{r}_D$ and $\mathbf{r}_E$ of points B, C, D and E can be expressed as

$$\mathbf{r}_B = \left[r_1 \cos\theta_1 + r_4 \cos\theta_4, \quad r_1 \sin\theta_1 + r_4 \sin\theta_4\right]^T \quad (11)$$

$$\mathbf{r}_C = \left[r_1 \cos\theta_1 + r_4 \cos\theta_4 + r_5 \cos\theta_5 , \quad r_1 \sin\theta_1 + r_4 \sin\theta_4 + r_5 \sin\theta_5\right]^T \quad (12)$$

$$\mathbf{r}_D = \left[r_1 \cos\theta_1 + r_7 \cos\theta_7, \quad y_C + r_7 \sin\theta_7\right]^T \quad (13)$$

$$\mathbf{r}_E = \left[r_1 \cos\theta_1 + l - r_{10} \cos\theta_{10}, \quad r_1 \sin\theta_1 + h + e - r_{10} \sin\theta_{10}\right]^T \quad (14)$$

where θ_4, θ_5 and θ_7 are the oriented angles of vectors $\mathbf{r}_4$, $\mathbf{r}_5$ and $\mathbf{r}_7$, respectively. The position vector $\mathbf{r}_A$ of point A is equal to $\mathbf{r}_2$ and thus, it is given by Eq. (5) for i = 2.

Therefore, the kinematic analysis of the proposed 10-bar long-dwell mechanism is formulated in the following up to the sixth-order, by considering each of the four assembled linkages separately, before to obtain the final formulation.

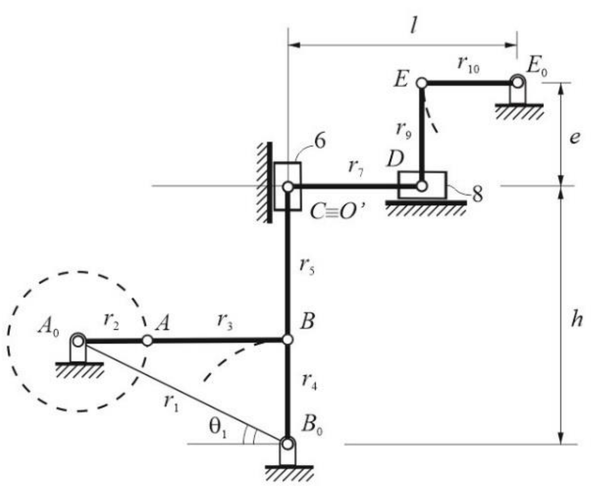

FIGURE.1: 10-BAR LONG-DWELL MECHANISM.

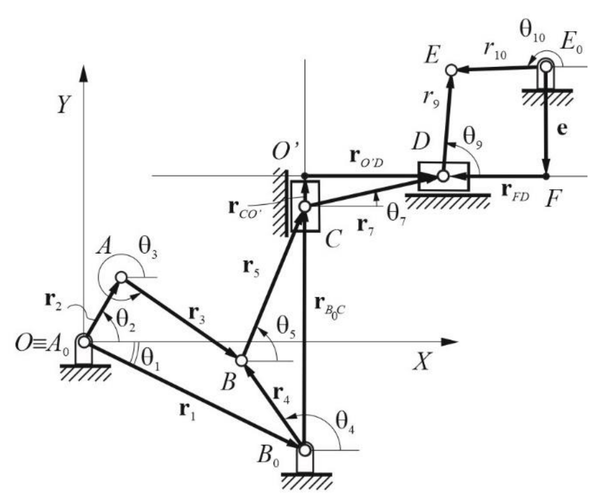

FIGURE 2: 10-BAR MECHANISM: VECTOR LOOPS.

The velocities, accelerations, jerks, snaps, crackles and pops of points *A*, *B*, *C*, *D* and *E* will be obtained, as function of θ_2, ω_2, α_2, φ_2 and ψ_2 of the driving crank, along with the angular velocities ω_3, ω_4, ω_5 and ω_6, accelerations α_3, α_4, α_5 and α_6, jerks φ_3 φ_4, φ_5 and φ_6, jounces ψ_3, ψ_4, ψ_5 and ψ_6, angular crackles ϕ_3, ϕ_4, ϕ_5 and ϕ_6 and pops γ_3, γ_4, γ_5 and γ_6.

2.1 Crank-rocker four-bar linkage

For the crank-rocker four-bar linkage of Fig. 1 and assuming to know the kinematic input data: θ_2, ω_2, α_2, φ_2, ψ_2, ϕ_2, γ_2 of the driving crank A_0A, the kinematic analysis is developed up to the sixth order with the aim to obtain angular and linear pop of point *B*. Thus, from the vector-loop equation (1), one has

$$\theta_4 = 2\tan^{-1}\frac{-\mathcal{B}+\sigma\sqrt{\mathcal{B}^2-C^2+\mathcal{A}^2}}{C-\mathcal{A}} \tag{15}$$

where σ is equal to ±1 according to a suitable assembly mode and the coefficients $\mathcal{A}$, $\mathcal{B}$ and C are obtained as function of the driving crank angle θ_2 by

$$\begin{aligned}
\mathcal{A} &= 2r_1 r_4\cos\theta_1 - 2r_2 r_4\cos\theta_2\\
\mathcal{B} &= 2r_1 r_4\sin\theta_1 - 2r_2 r_4\sin\theta_2\\
C &= r_1^2+r_2^2+r_4^2-r_3^2-2r_1 r_2\left(\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\right)
\end{aligned} \tag{16}$$

Moreover, Eq. (1) can be solved with respect to θ_3 as

$$\theta_3 = \tan^{-1}\frac{r_1\sin\theta_1 + r_4\sin\theta_4 - r_2\sin\theta_2}{r_1\cos\theta_1 + r_4\cos\theta_4 - r_2\cos\theta_2} \tag{17}$$

From the first time-derivative of Eq. (1), the angular velocities ω_3 and ω_4 are obtained as function of the driving angular velocity ω_2 and the crank angle θ_2 by

$$\omega_4 = \frac{r_2\sin\left(\theta_2-\theta_3\right)}{r_4\sin\left(\theta_4-\theta_3\right)}\omega_2 \tag{18}$$

$$\omega_3 = \frac{r_2\sin\left(\theta_2-\theta_4\right)}{r_3\sin\left(\theta_4-\theta_3\right)}\omega_2 \tag{19}$$

where the angles θ_3 and θ_4 are given by Eqs. (17) - (19).

The velocity vector $\mathbf{v}_B$ of point *B*, as first-time derivative of Eq. (11), takes the form

$$\mathbf{v}_B = \omega_4\left[-r_4\sin\theta_4, \quad r_4\cos\theta_4\right]^T \tag{20}$$

where ω_4 and θ_4 are expressed by the Eqs. (18) – (19) and (15) as function of θ_2 and ω_2.

Similarly, by the knowledge of the angular acceleration α_2 of the driving crank A_0A and developing the second-time derivative of Eq. (1), the angular accelerations α_3 and α_4 are given by

$$\alpha_3 = \frac{-r_2\,\alpha_2\sin\left(\theta_4-\theta_2\right)+r_2\,\omega_2^2\cos\left(\theta_4-\theta_2\right)+}{r_3\sin\left(\theta_4-\theta_3\right)} \frac{+r_3\,\omega_3^2\cos\left(\theta_4-\theta_3\right)-r_4\,\omega_4^2}{r_3\sin\left(\theta_4-\theta_3\right)} \tag{21}$$

$$\alpha_4 = \frac{-r_2\,\alpha_2\sin\left(\theta_3-\theta_2\right)+r_2\,\omega_2^2\cos\left(\theta_3-\theta_2\right)+}{r_4\sin\left(\theta_4-\theta_3\right)} \frac{-r_4\,\omega_4^2\cos\left(\theta_4-\theta_3\right)+r_3\,\omega_3^2}{r_4\sin\left(\theta_4-\theta_3\right)} \tag{22}$$

and, in turn, the acceleration vector $\mathbf{a}_B$ of point *B* is given by

$$\begin{aligned}\mathbf{a}_B = \big[&-r_4\,\alpha_4\sin\theta_4 - r_4\,\omega_4^2\cos\theta_4,\\ &r_4\,\alpha_4\cos\theta_4 - r_4\,\omega_4^2\sin\theta_4\big]^T\end{aligned} \tag{23}$$

By the angular jerk φ_2 and developing the third-time derivative of Eq. (1), the angular jerks φ_3 and φ_4 are given by

$$\varphi_3 = -\frac{A_1 + B_1\tan\theta_4}{r_3\left(\cos\theta_3\tan\theta_4 - \sin\theta_3\right)} \tag{24}$$

$$\varphi_4 = \frac{\varphi_3 r_3\cos\theta_3 + B_1}{r_4\cos\theta_4} \tag{25}$$

where

$$\begin{aligned}
A_1 = {} & r_2\,\omega_2^3\sin\theta_2 - 3r_2\,\omega_2\alpha_2\cos\theta_2 - r_2\,\varphi_2\sin\theta_2 +\\
& + r_3\,\omega_3^3\sin\theta_3 - 3r_3\,\omega_3\alpha_3\cos\theta_3 +\\
& - r_4\,\omega_4^3\sin\theta_4 + 3r_4\,\omega_4\alpha_4\cos\theta_4
\end{aligned} \tag{26}$$

$$\begin{aligned}
B_1 = {} & -r_2\,\omega_2^3\cos\theta_2 - 3r_2\,\omega_2\alpha_2\sin\theta_2 + r_2\,\varphi_2\cos\theta_2 +\\
& - r_3\,\omega_3^3\cos\theta_3 - 3r_3\,\omega_3\alpha_3\sin\theta_3 +\\
& + r_4\,\omega_4^3\cos\theta_4 + 3r_4\,\omega_4\alpha_4\sin\theta_4
\end{aligned} \tag{27}$$

as function of the angles θ_2, θ_3 and θ_4, the angular velocities ω_2, ω_3 and ω_4, the angular accelerations α_2, α_3 and α_4, along with the angular jerk φ_2.

The jerk vector $\mathbf{J}_B$ of point *B* takes the form

$$\mathbf{J}_B = \begin{bmatrix} -r_4\,\varphi_4\sin\theta_4 - 3r_4\,\omega_4\,\alpha_4\cos\theta_4 + r_4\,\omega_4^3\sin\theta_4 \\ r_4\,\varphi_4\cos\theta_4 - 3r_4\,\omega_4\,\alpha_4\sin\theta_4 - r_4\,\omega_4^3\cos\theta_4 \end{bmatrix} \tag{28}$$

as function of the driven angle θ_4 and the velocity ω_4, acceleration α_4 and jerk φ_4.

Thus, supposing to know the angular jounce ψ_2 and developing the fourth-time derivative of Eq. (1), the angular jounce ψ_3 and ψ_4 takes can be expressed as

$$\psi_3 = -\frac{A_2 + B_2\tan\theta_4}{r_3\left(\cos\theta_3\tan\theta_4 - \sin\theta_3\right)} \qquad (29)$$

$$\psi_4 = \frac{\psi_3 r_3\cos\theta_3 + B_2}{r_4\cos\theta_4} \qquad (30)$$

where

$$\begin{aligned} A_2 = {} & 6r_2\,\omega_2^2\alpha_2\sin\theta_2 + r_2\,\omega_2^4\cos\theta_2 - r_2\,\alpha_2^2\cos\theta_2 - \\ & +4r_2\,\omega_2\varphi_2\cos\theta_2 - r_2\,\psi_2\sin\theta_2 + 6r_3\,\omega_3^2\alpha_3\sin\theta_3 + \\ & +r_3\,\omega_3^4\cos\theta_3 - r_3\,\alpha_3^2\cos\theta_3 - 4r_3\,\omega_3\varphi_3\cos\theta_3 + \\ & +6r_4\,\omega_4^2\alpha_4\sin\theta_4 + r_4\,\omega_4^4\cos\theta_4 - r_4\,\alpha_4^2\cos\theta_4 + \\ & -4r_4\,\omega_4\varphi_4\cos\theta_4 \end{aligned} \qquad (31)$$

$$\begin{aligned} B_2 = {} & -6r_2\,\omega_2^2\alpha_2\cos\theta_2 + r_2\,\omega_2^4\sin\theta_2 - r_2\,\alpha_2^2\sin\theta_2 - \\ & +4r_2\,\omega_2\varphi_2\sin\theta_2 + r_2\,\psi_2\cos\theta_2 - 6r_3\,\omega_3^2\alpha_3\cos\theta_3 + \\ & +r_3\,\omega_3^4\sin\theta_3 - r_3\,\alpha_3^2\sin\theta_3 - 4r_3\,\omega_3\varphi_3\sin\theta_3 + \\ & +6r_4\,\omega_4^2\alpha_4\cos\theta_4 + r_4\,\omega_4^4\sin\theta_4 + r_4\,\alpha_4^2\sin\theta_4 + \\ & +4r_4\,\omega_4\varphi_4\sin\theta_4 \end{aligned} \qquad (32)$$

Thus, the jounce, or snap, vector $\mathbf{S}_B$ of point B takes the form

$$\mathbf{S}_B = \begin{bmatrix} -r_4\,\psi_4\sin\theta_4 - 4r_4\,\omega_4\,\varphi_4\cos\theta_4 - 3r_4\,\alpha_4^2\cos\theta_4 + \\ +3r_4\,\omega_4^2\,\alpha_4\sin\theta_4 + 3r_4\,\omega_4^2\sin\theta_4 + r_4\,\omega_4^4\cos\theta_4 \\ \\ r_4\,\psi_4\cos\theta_4 - 4r_4\,\omega_4\,\varphi_4\sin\theta_4 - 3r_4\,\alpha_4^2\sin\theta_4 - \\ +3r_4\,\omega_4^2\,\alpha_4\cos\theta_4 - 3r_4\,\omega_4^2\cos\theta_4 + r_4\,\omega_4^4\sin\theta_4 \end{bmatrix} \qquad (33)$$

Thus, supposing to know the angular crackle ϕ_2 and developing the fifth-time derivative of Eq. (1), the angular crackle ϕ_3 and ϕ_4 takes can be expressed as

$$\phi_3 = -\frac{A_3 + B_3\tan\theta_4}{r_3\left(\cos\theta_3\tan\theta_4 - \sin\theta_3\right)} \qquad (34)$$

$$\phi_4 = \frac{\phi_3 r_3\cos\theta_3 + B_3}{r_4\cos\theta_4} \qquad (35)$$

where

$$\begin{aligned} A_3 = {} & -r_2\,\omega_2^5\sin\theta_2 + 10r_2\,\alpha_2\omega_2^3\cos\theta_2 + 15r_2\,\alpha_2^2\omega_2\sin\theta_2 + \\ & +10r_2\,\omega_2^2\varphi_2\sin\theta_2 - 10r_2\,\alpha_2\varphi_2\cos\theta_2 - 5r_2\,\omega_2\psi_2\cos\theta_2 + \\ & -r_2\,\phi_2\sin\theta_2 - r_3\,\omega_3^5\sin\theta_3 + 10r_3\,\alpha_3\omega_3^3\cos\theta_3 + \\ & +15r_3\,\alpha_3^2\omega_3\sin\theta_3 + 10r_3\,\omega_3^2\varphi_3\sin\theta_3 - \\ & -10r_3\,\alpha_3\varphi_3\cos\theta_3 - 5r_3\,\omega_3\psi_3\cos\theta_3 + r_4\,\omega_4^5\sin\theta_4 + \\ & -10r_4\,\alpha_4\omega_4^3\cos\theta_4 - 15r_4\,\alpha_4^2\omega_4\sin\theta_4 - \\ & -10r_4\,\omega_4^2\varphi_4\sin\theta_4 + 10r_4\,\alpha_4\varphi_4\cos\theta_4 + 5r_4\,\omega_4\psi_4\cos\theta_4 \end{aligned} \qquad (36)$$

$$\begin{aligned} B_3 = {} & r_2\,\omega_2^5\cos\theta_2 + 10r_2\,\alpha_2\omega_2^3\sin\theta_2 - 15r_2\,\alpha_2^2\omega_2\cos\theta_2 - \\ & +10r_2\,\omega_2^2\varphi_2\cos\theta_2 - 10r_2\,\alpha_2\varphi_2\sin\theta_2 - 5r_2\,\omega_2\psi_2\sin\theta_2 + \\ & +r_2\,\phi_2\cos\theta_2 + r_3\,\omega_3^5\cos\theta_3 + 10r_3\,\alpha_3\omega_3^3\cos\theta_3 + \\ & +15r_3\,\alpha_3^2\omega_3\cos\theta_3 - 10r_3\,\omega_3^2\varphi_3\cos\theta_3 - \\ & -10r_3\,\alpha_3\varphi_3\sin\theta_3 - 5r_3\,\omega_3\psi_3\sin\theta_3 - r_4\,\omega_4^5\cos\theta_4 - \\ & +10r_4\,\alpha_4\omega_4^3\sin\theta_4 + 15r_4\,\alpha_4^2\omega_4\cos\theta_4 + \\ & +10r_4\,\omega_4^2\varphi_4\cos\theta_4 + 10r_4\,\alpha_4\varphi_4\sin\theta_4 + 5r_4\,\omega_4\psi_4\sin\theta_4 \end{aligned} \qquad (37)$$

Therefore, the crackle vector $\mathbf{C}_B$ of point B takes the form

$$\mathbf{C}_B = \begin{bmatrix} -r_4\,\phi_4\sin\theta_4 + 10r_4\,\omega_4^3\,\alpha_4\cos\theta_4 - r_4\,\omega_4^5\sin\theta_4 ++ \\ 15r_4\,\omega_4\,\alpha_4^2\sin\theta_4 + 10r_4\,\omega_4^2\varphi_4\sin\theta_4 + \\ -10r_4\,\alpha_4\varphi_4\cos\theta_4 - 5r_4\,\omega_4\psi_4\cos\theta_4 \\ \\ r_4\,\phi_4\cos\theta_4 + 10r_4\,\omega_4^3\,\alpha_4\sin\theta_4 + r_4\,\omega_4^5\cos\theta_4 + \\ -15r_4\,\omega_4\,\alpha_4^2\cos\theta_4 - 10r_4\,\omega_4^2\varphi_4\cos\theta_4 + \\ -10r_4\,\alpha_4\varphi_4\sin\theta_4 - 5r_4\,\omega_4\psi_4\sin\theta_4 \end{bmatrix} \qquad (38)$$

Finally, supposing to know the angular pop γ_2 and developing the sixth-time derivative of Eq. (1), the angular pop γ_3 and γ_4 can be expressed as

$$\gamma_3 = -\frac{A_4 + B_4\tan\theta_4}{r_3\left(\cos\theta_3\tan\theta_4 - \sin\theta_3\right)} \qquad (39)$$

$$\gamma_4 = \frac{\gamma_3 r_3\cos\theta_3 + B_4}{r_4\cos\theta_4} \qquad (40)$$

where

$$\begin{aligned} A_4 = {} & 15r_2\,\alpha_2^3\sin\theta_2 - 10r_2\varphi_2^2\cos\theta_2 - 15r_2\,\omega_2^4\alpha_2\sin\theta_2 + \\ & -r_2\,\omega_2^6\cos\theta_2 ++20r_2\,\omega_2^3\varphi_2\cos\theta_2 + 15r_2\,\omega_2^2\psi_2\sin\theta_2 + \\ & -45r_2\,\omega_2^2\alpha_2^2\cos\theta_2 - 15r_2\,\alpha_2\psi_2\cos\theta_2 - 6r_2\,\omega_2\phi_2\cos\theta_2 + \\ & +60r_2\,\omega_2\alpha_2\varphi_2\sin\theta_2 - r_2\,\gamma_2\sin\theta_2 + 15r_3\,\alpha_3^3\sin\theta_3 + \\ & -10r_3\varphi_3^2\cos\theta_3 - 15r_3\,\omega_3^4\alpha_3\sin\theta_3 + \\ & -r_3\,\omega_3^6\cos\theta_3 + 20r_3\,\omega_3^3\varphi_3\cos\theta_3 + 15r_3\,\omega_3^2\psi_3\sin\theta_3 + \\ & +45r_3\,\omega_3^2\alpha_3^2\cos\theta_3 - 15r_3\,\alpha_3\psi_3\cos\theta_3 - 6r_3\,\omega_3\phi_3\cos\theta_3 + \\ & +60r_3\,\omega_3\alpha_3\varphi_3\sin\theta_3 - 15r_4\,\alpha_4^3\sin\theta_4 + 10r_4\varphi_4^2\cos\theta_4 + \\ & +15r_4\,\omega_4^4\alpha_4\sin\theta_4 + r_4\,\omega_4^6\cos\theta_4 - 20r_4\,\omega_4^3\varphi_4\cos\theta_4 + \\ & -15r_4\,\omega_4^2\psi_4\sin\theta_4 - 45r_4\,\omega_4^2\alpha_4^2\cos\theta_4 + 15r_4\,\alpha_4\psi_4\cos\theta_4 + \\ & +6r_4\,\omega_4\phi_4\cos\theta_4 - 60r_4\,\omega_4\alpha_4\varphi_4\sin\theta_4 \end{aligned} \qquad (41)$$

$$
\begin{aligned}
B_4 = &-15r_2\,\alpha_2^3\cos\theta_2 - 10r_2\varphi_2^2\sin\theta_2 + 15r_2\,\omega_2^4\alpha_2\cos\theta_2 + \\
&- r_2\,\omega_2^6\sin\theta_2 + 20r_2\,\omega_2^3\varphi_2\sin\theta_2 - 15r_2\,\omega_2^2\psi_2\cos\theta_2 + \\
&+ 45r_2\,\omega_2^2\alpha_2^2\sin\theta_2 - 15r_2\,\alpha_2\psi_2\sin\theta_2 - 6r_2\,\omega_2\phi_2\sin\theta_2 + \\
&- 60r_2\,\omega_2\alpha_2\varphi_2\cos\theta_2 + r_2\,\gamma_2\cos\theta_2 - 15r_3\,\alpha_3^3\cos\theta_3 + \\
&- 10r_3\varphi_3^2\sin\theta_3 + 15r_3\,\omega_3^4\alpha_3\cos\theta_3 \\
&- r_3\,\omega_3^6\sin\theta_3 + 20r_3\,\omega_3^3\varphi_3\sin\theta_3 - 15r_3\,\omega_3^2\psi_3\cos\theta_3 + \\
&+ 45r_3\,\omega_3^2\alpha_3^2\sin\theta_3 - 15r_3\,\alpha_3\psi_3\sin\theta_3 - 6r_3\,\omega_3\phi_3\sin\theta_3 + \\
&- 60r_3\,\omega_3\alpha_3\varphi_3\cos\theta_3 + 15r_4\,\alpha_4^3\cos\theta_4 + 10r_4\varphi_4^2\sin\theta_4 + \\
&- 15r_4\,\omega_4^4\alpha_4\cos\theta_4 + r_4\,\omega_4^6\sin\theta_4 - 20r_4\,\omega_4^3\varphi_4\sin\theta_4 + \\
&+ 15r_4\,\omega_4^2\psi_4\cos\theta_4 - 45r_4\,\omega_4^2\alpha_4^2\sin\theta_4 + 15r_4\,\alpha_4\psi_4\sin\theta_4 + \\
&+ 6r_4\,\omega_4\phi_4\sin\theta_4 + 60r_4\,\omega_4\alpha_4\varphi_4\cos\theta_4
\end{aligned}
\tag{42}
$$

Therefore, the pop vector $\mathbf{P}_B$ of point B is obtained in the form

$$
\mathbf{P}_B = \begin{bmatrix}
-r_4\,\gamma_4\sin\theta_4 - 10r_4\,\varphi_4^2\cos\theta_4 - r_4\,\omega_4^6\cos\theta_4 + \\
+15r_4\,\alpha_4^3\sin\theta_4 - 15r_4\,\omega_4^4\alpha_4\sin\theta_4 + \\
+20r_4\,\omega_4^3\varphi_4\cos\theta_4 + 15r_4\,\omega_4^2\psi_4\sin\theta_4 + \\
+45r_4\,\omega_4^2\alpha_4^2\cos\theta_4 - 15r_4\,\alpha_4\psi_4\cos\theta_4 + \\
-6r_4\,\omega_4\phi_4\cos\theta_4 + 60r_4\,\omega_4\alpha_4\varphi_4\sin\theta_4 \\
\\
r_4\,\gamma_4\cos\theta_4 - 10r_4\,\varphi_4^2\sin\theta_4 - r_4\,\omega_4^6\sin\theta_4 + \\
-15r_4\,\alpha_4^3\cos\theta_4 + 15r_4\,\omega_4^4\alpha_4\cos\theta_4 + \\
+20r_4\,\omega_4^3\varphi_4\sin\theta_4 - 15r_4\,\omega_4^2\psi_4\cos\theta_4 + \\
+45r_4\,\omega_4^2\alpha_4^2\sin\theta_4 - 15r_4\,\alpha_4\psi_4\sin\theta_4 + \\
-6r_4\,\omega_4\phi_4\sin\theta_4 - 60r_4\,\omega_4\alpha_4\varphi_4\cos\theta_4
\end{bmatrix}
\tag{43}
$$

2.2 Centered slider-crank mechainsm

For the slider-crank/rocker mechanism, referring to Fig. 1 and assuming to know the kinematic input data: θ_4, ω_4, α_4, φ_4, ψ_4, ϕ_4, γ_4 of the driving link B_0B, which coincides in this case with the driven link of the four-bar linkage, the kinematic analysis is developed up to the sixth order with the aim to obtain the pop of point C.

Thus, from the vector-loop equation (2), one has

$$
\theta_5 = \cos^{-1}\left(\frac{x_C - x_B}{r_5}\right) \tag{44}
$$

and differentiating a first time, the angular velocity ω_5 takes the form

$$
\omega_5 = \frac{v_{Bx}}{r_5\sin\theta_5} \tag{45}
$$

and, in turn, the velocity vector $\mathbf{v}_C$ of point C is given by

$$
\mathbf{v}_C = \begin{bmatrix} 0, & v_{By} + r_5\,\omega_5\cos\theta_5 \end{bmatrix}^T \tag{46}
$$

as function of the kinematic input data.

From the second-time derivative of Eq. (2), the angular acceleration α_5 of the coupler link BC of the centered slider-crank mechanism and the acceleration vector $\mathbf{a}_C$ of point C, which revolute kinematic pair joins the piston 6 to the CD coupler link of the orthogonal Cardan mechanism, can be expressed as follows

$$
\alpha_5 = \frac{a_{Bx} - r_5\,\omega_5^2\cos\theta_5}{r_5\sin\theta_5} \tag{47}
$$

$$
\mathbf{a}_C = \begin{bmatrix} 0, & a_{By} + r_5\alpha_5\cos\theta_5 - r_5\omega_5^2\sin\theta_5 \end{bmatrix}^T \tag{48}
$$

and in turn, from the third-time derivative of Eq. (2), the angular jerk φ_5 and the jerk vector $\mathbf{J}_C$ of point C take the following expressions

$$
\varphi_5 = \frac{J_{Bx} - 3r_5\,\omega_5\,\alpha_5\cos\theta_5 + r_5\,\omega_5^3\sin\theta_5}{r_5\sin\theta_5} \tag{49}
$$

$$
\mathbf{J}_C = \begin{bmatrix} 0 \\ J_{By} + r_5\varphi_5\cos\theta_5 - 3r_5\,\omega_5\,\alpha_5\sin\theta_5 - r_5\,\omega_5^3\sin\theta_5 \end{bmatrix} \tag{50}
$$

where the X-component is equal to zero since point C moves along the Y-axis.

Similarly, from the fourth-time derivative of Eq. (2), the angular jounce or snap ψ_5 and the snap vector $\mathbf{S}_C$ of point C are given by

$$
\psi_5 = \frac{S_{Bx} - 4r_5\,\omega_5\,\varphi_5\cos\theta_5 + 6r_5\,\omega_5^2\alpha_5\sin\theta_5 +}{r_5\sin\theta_5}
\frac{-3r_5\,\alpha_5\cos\theta_5 + r_5\,\omega_5^4\cos\theta_5}{r_5\sin\theta_5} \tag{51}
$$

$$
\mathbf{S}_C = \begin{bmatrix} 0 \\ \\ S_{By} + r_5\psi_5\cos\theta_5 - 4r_5\,\omega_5\,\varphi_5\sin\theta_5 + \\ -6r_5\,\omega_5^2\alpha_5\cos\theta_5 - 3r_5\,\alpha_5\sin\theta_5 + r_5\,\omega_5^4\sin\theta_5 \end{bmatrix} \tag{52}
$$

Likewise, from the fifth-time derivative of Eq. (2), the angular crackle ϕ_5 and the crackle vector $\mathbf{C}_C$ of point C can be expressed in the following form

$$\phi_5 = \frac{C_{Bx} + 10 r_5 \omega_5^2 \varphi_5 \sin\theta_5 - 10 r_5 \alpha_5 \varphi_5 \cos\theta_5}{r_5 \sin\theta_5} + \frac{10 r_5 \omega_5 \alpha_5^2 \sin\theta_5 + 10 r_5 \omega_5^3 \alpha_5 \cos\theta_5}{r_5 \sin\theta_5} + \frac{-5 r_5 \omega_5 \psi_5 \cos\theta_5 - r_5 \omega_5^5 \sin\theta_5}{r_5 \sin\theta_5} \tag{53}$$

$$\mathbf{C}_C = \begin{bmatrix} 0 \\ C_B y + r_5 \phi_5 \cos\theta_5 + r_5 \omega_5^5 \sin\theta_5 - 15 r_5 \omega_5 \alpha_5^2 \cos\theta_5 + \\ +10 r_5 \omega_5^3 \alpha_5 \sin\theta_5 - 10 r_5 \omega_5^2 \varphi_5 \sin\theta_5 - \\ +10 r_5 \omega_5 \alpha_5 \varphi_5 \sin\theta_5 - 5 r_5 \omega_5 \psi_5 \sin\theta_5 \end{bmatrix} \tag{54}$$

Finally, the angular pop γ_5 and the pop vector $\mathbf{P}_C$ of point C are given by

$$\gamma_5 = \frac{P_{Bx} + 15 r_5 \alpha_5^3 \sin\theta_5 - 10 r_5 \varphi_5^2 \cos\theta_5 - r_5 \omega_5^6 \cos\theta_5}{r_5 \sin\theta_5} + \frac{-15 r_5 \omega_5^4 \alpha_5 \sin\theta_5 + 20 r_5 \varphi_5 \omega_5^3 \cos\theta_5 + 15 r_5 \omega_5^2 \psi_5 \sin\theta_5}{r_5 \sin\theta_5} + \frac{45 r_5 \omega_5^2 \alpha_5 \cos\theta_5 - 15 r_5 \alpha_5 \psi_5 \cos\theta_5}{r_5 \sin\theta_5} + \frac{-6 r_5 \phi_5 \omega_5 \cos\theta_5 + 60 r_5 \alpha_5 \omega_5 \varphi_5 \sin\theta_5}{r_5 \sin\theta_5} \tag{55}$$

$$\mathbf{P}_C = \begin{bmatrix} 0 \\ P_{By} + 15 r_5 \alpha_5^3 \cos\theta_5 - 10 r_5 \varphi_5^2 \sin\theta_5 - r_5 \omega_5^6 \sin\theta_5 + \\ +15 r_5 \omega_5^4 \alpha_5 \cos\theta_5 + 20 r_5 \varphi_5 \omega_5^3 \sin\theta_5 + r_5 \gamma_5 \cos\theta_5 + \\ +15 r_5 \omega_5^2 \psi_5 \cos\theta_5 + 45 r_5 \omega_5^2 \alpha_5 \sin\theta_5 + \\ -15 r_5 \alpha_5 \psi_5 \sin\theta_5 - 6 r_5 \phi_5 \omega_5 \sin\theta_5 + \\ -60 r_5 \alpha_5 \omega_5 \varphi_5 \cos\theta_5 \end{bmatrix} \tag{56}$$

2.3 Orthogonal Cardan mechanism

For the orthogonal Cardan mechanism, still referring to Fig. 1 and assuming to know the kinematic input data: y_C, v_{Cy}, a_{Cy}, J_{Cy}, S_{Cy} and P_{Cy} which correspond respectively to the position, velocity, acceleration, jerk, jounce or snap, crackle and pop of point C of piston 6 that is also the driving member of the Cardan mechanism, from the vector-loop equation (3), one has

$$\theta_7 = \sin^{-1}\left(\frac{r_5 - y_C}{r_7}\right) \tag{57}$$

and, in turn, the angular velocity ω_7 of the coupler link r_7 and the velocity vector $\mathbf{v}_D$ of point D of the dwell piston 8, are expressed as follows

$$\omega_7 = -\frac{v_{Cy}}{r_7 \cos\theta_7} \tag{58}$$

$$\mathbf{v}_D = \left[-r_7 \omega_7 \sin\theta_7, \quad 0\right]^T \tag{59}$$

The angular acceleration α_7 and the acceleration vector $\mathbf{a}_D$ of point D are given by

$$\alpha_7 = \frac{r_7 \omega_7^2 \sin\theta_7 - a_{Cy}}{r_7 \cos\theta_7} \tag{60}$$

$$\mathbf{a}_D = \left[-r_7 \alpha_7 \sin\theta_7 - r_7 \omega_7^2 \cos\theta_7, \quad 0\right]^T \tag{61}$$

and, consequently, the angular jerk φ_7 and the jerk vector $\mathbf{J}_D$ take the expressions

$$\varphi_7 = \frac{3 r_7 \omega_7 \alpha_7 \sin\theta_7 + r_7 \omega_7^3 \cos\theta_7 - J_{Cy}}{r_7 \cos\theta_7} \tag{62}$$

$$\mathbf{J}_D = \begin{bmatrix} -r_7 \varphi_7 \sin\theta_7 - 3 r_7 \omega_7 \alpha_7 \cos\theta_7 + r_7 \omega_7^3 \sin\theta_7 \\ 0 \end{bmatrix} \tag{63}$$

similarly, the angular jounce ψ 7 and the snap vector $\mathbf{S}_D$ of point D can be expressed as follows

$$\psi_7 = \frac{4 r_7 \omega_7 \varphi_7 \sin\theta_7 + 3 r_7 \alpha_7^2 \sin\theta_7 + 6 r_7 \omega_7^2 \alpha_7 \cos\theta_7 +}{r_7 \cos\theta_7} \; \frac{-r_7 \omega_7^4 \sin\theta_7 - S_{Cy}}{r_7 \cos\theta_7} \tag{64}$$

$$\mathbf{S}_D = \begin{bmatrix} -r_7 \psi_7 \sin\theta_7 - 4 r_7 \omega_7 \varphi_7 \cos\theta_7 - 3 r_7 \alpha_7^2 \cos\theta_7 + \\ +6 r_7 \omega_7^2 \alpha_7 \sin\theta_7 + r_7 \omega_7^4 \cos\theta_7 \\ \\ 0 \end{bmatrix} \tag{65}$$

where the Y-component of $\mathbf{S}_D$ is equal to zero since the dwell piston 8 translates along the X-axis.

Similarly, the angular crackle ϕ_7 and the crackle vector $\mathbf{C}_D$ of point D are given by

$$\phi_7 = \frac{C_{Cy} + 10r_7\,\omega_7^2\,\varphi_7\cos\theta_7 + 10r_7\,\alpha_7\,\varphi_7\sin\theta_7}{r_7\cos\theta_7} + \frac{15r_7\,\omega_7\alpha_7^2\cos\theta_7 - 10r_7\,\omega_7^3\,\alpha_7\sin\theta_7}{r_7\cos\theta_7} + \frac{5r_7\,\omega_7\,\psi_7\sin\theta_7 - r_7\,\omega_7^5\cos\theta_7}{r_7\cos\theta_7} \tag{66}$$

$$\mathbf{C}_C = \begin{bmatrix} 0 \\ 10r_7\,\omega_7^2\,\varphi_7\sin\theta_7 - 10r_7\,\alpha_7\,\varphi_7\cos\theta_7 + \\ +15r_7\,\omega_7\alpha_7^2\sin\theta_7 + 10r_7\,\omega_7^3\,\alpha_7\cos\theta_7 + \\ +5r_7\,\omega_7\,\psi_7\cos\theta_7 - r_7\,\omega_7^5\sin\theta_7 - r_7\,\phi_7\sin\theta_7 \end{bmatrix} \tag{67}$$

Finally, the angular pop γ_7 and the pop vector $\mathbf{P}_D$ of point D are

$$\gamma_7 = \frac{-P_{Cy} + 15r_7\,\alpha_7^3\cos\theta_7 + 10r_7\,\varphi_7^2\sin\theta_7 + r_7\,\omega_7^6\sin\theta_7 +}{r_7\cos\theta_7} \frac{-15r_7\omega_7^4\,\alpha_7\cos\theta_7 + 20r_7\varphi_7\omega_7^3\sin\theta_7 +}{r_7\cos\theta_7} \frac{+15r_7\omega_7^2\,\psi_7\cos\theta_7 - 45r_7\omega_7^2\,\alpha_7^2\sin\theta_7 +}{r_7\cos\theta_7} \frac{+15\,r_7\alpha_7\,\psi_7\sin\theta_7 + 6r_7\phi_7\omega_7\sin\theta_7 + 60r_7\alpha_7\omega_7\varphi_7\cos\theta_7}{r_7\cos\theta_7} \tag{68}$$

$$\mathbf{P}_C = \begin{bmatrix} 0 \\ 15r_7\,\alpha_7^3\sin\theta_7 - 10r_7\,\varphi_7^2\cos\theta_7 - r_7\,\omega_7^6\cos\theta_7 - \\ +15r_7\omega_7^4\,\alpha_7\sin\theta_7 + 20r_7\varphi_7\omega_7^3\cos\theta_7 + r_7\psi_7\sin\theta_7 \\ +15r_7\omega_7^2\,\psi_7\sin\theta_7 + 45r_7\omega_7^2\,\alpha_7^2\cos\theta_7 + \\ +15\,r_7\alpha_7\,\psi_7\cos\theta_7 - 6r_7\phi_7\omega_7\cos\theta_7 + \\ +60r_7\alpha_7\omega_7\varphi_7\sin\theta_7 \end{bmatrix} \tag{69}$$

2.4 Offset slider-rocker mechanism

For the offset- slider- rocker mechanism, referring to Fig. 1 and assuming to know the kinematic input data: x_D, v_{Dx}, a_{Dx}, J_{Dx}, S_{Dx} and P_{Dx} which correspond respectively to the position, velocity, acceleration, jerk, jounce or snap, crackle and pop of point D of piston 8 that is also the driving member of the offset slider-rocker mechanism, from the vector-loop equation (4), one has

$$\theta_9 = 2\tan^{-1}\frac{-\mathcal{E} + \sigma\sqrt{\mathcal{E}^2 - \mathcal{F}^2 + \mathcal{D}^2}}{\mathcal{F} - \mathcal{D}} \tag{70}$$

where σ is equal to ±1 according to a suitable assembly mode and the coefficients $\mathcal{D}$, $\mathcal{E}$ and $\mathcal{F}$ as function of x_D by

$$\begin{aligned} \mathcal{D} &= 2r_9\left(x_{E0} - x_D\right) \\ \mathcal{E} &= 2r_9\,e \\ \mathcal{F} &= -r_9^2 + r_{10}^2 - \left(x_{E0} - x_D\right)^2 - e^2 \end{aligned} \tag{71}$$

$$\theta_{10} = \sin^{-1}\left(\frac{e - r_9\sin\theta_9}{r_{10}}\right) \tag{72}$$

and differentiating, ω_9 and ω_{10} take the form

$$\omega_9 = \frac{v_{Dx}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{73}$$

$$\omega_{10} = -\frac{r_9\cos\theta_9}{r_{10}\cos\theta_{10}}\,\omega_9 \tag{74}$$

and, in turn, the velocity vector $\mathbf{v}_E$ of point E is given by

$$\mathbf{v}_E = \left[r_{10}\,\omega_{10}\sin\theta_{10},\quad -r_{10}\,\omega_{10}\cos\theta_{10}\right]^T \tag{75}$$

as function of the kinematic input data.

From the second-time derivative of Eq. (4), the angular accelerations α_9, α_{10} and the acceleration vector $\mathbf{a}_E$ can be expressed as

$$\alpha_9 = \frac{A_5 + B_5\tan\theta_{10}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{76}$$

$$\alpha_{10} = -\frac{B_5 + r_9\alpha_9\cos\theta_9}{r_{10}\cos\theta_{10}} \tag{77}$$

$$\begin{aligned} \mathbf{a}_E = \big[& r_{10}\,\alpha_{10}\sin\theta_{10} + r_{10}\,\omega_{10}^2\cos\theta_{10}, \\ & -r_{10}\,\alpha_{10}\cos\theta_{10} + r_{10}\,\omega_{10}^2\sin\theta_{10}\big]^T \end{aligned} \tag{78}$$

where

$$\begin{aligned} A_5 &= a_{Dx} - r_9\omega_9^2\cos\theta_9 - r_{10}\omega_{10}^2\cos\theta_{10} \\ B_5 &= -r_9\omega_9^2\sin\theta_9 - r_{10}\omega_{10}^2\sin\theta_{10} \end{aligned} \tag{79}$$

and in turn, from the third-time derivative of Eq. (2), the angular jerks φ_9, φ_{10} and the jerk vector $\mathbf{J}_E$ of point E take the following expressions

$$\varphi_9 = \frac{A_6 + B_6\tan\theta_{10}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{80}$$

$$\varphi_{10} = -\frac{B_6 + r_9\varphi_9 \cos\theta_9}{r_{10}\cos\theta_{10}} \tag{81}$$

$$\mathbf{J}_E = \begin{bmatrix} r_{10}\varphi_{10}\sin\theta_{10} + 3r_{10}\omega_{10}\alpha_{10}\cos\theta_{10} - r_{10}\omega_{10}^3\sin\theta_{10} \\ r_{10}\varphi_{10}\cos\theta_{10} + 3r_{10}\omega_{10}\alpha_{10}\sin\theta_{10} + r_{10}\omega_{10}^3\cos\theta_{10} \end{bmatrix} \tag{82}$$

where

$$\begin{aligned} A_6 = J_{Dx} + r_9\omega_9^3\sin\theta_9 + r_{10}\omega_{10}^3\sin\theta_{10} - 3r_9\alpha_9\cos\theta_9 + \\ -3r_{10}\omega_{10}\alpha_{10}\cos\theta_{10} \end{aligned} \tag{83}$$

$$\begin{aligned} B_6 = -r_9\omega_9^3\cos\theta_9 - r_{10}\omega_{10}^3\cos\theta_{10} - 3r_9\alpha_9\sin\theta_9 + \\ -3r_{10}\omega_{10}\alpha_{10}\sin\theta_{10} \end{aligned} \tag{84}$$

Similarly, the angular jounce ψ_9, ψ_{10} and the snap vector $\mathbf{S}_E$ of point E are given by

$$\psi_9 = \frac{A_7 + B_7\tan\theta_{10}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{85}$$

$$\psi_{10} = -\frac{B_7 + r_9\psi_9\cos\theta_9}{r_{10}\cos\theta_{10}} \tag{86}$$

$$\mathbf{S}_E = \begin{bmatrix} r_{10}\psi_{10}\sin\theta_{10} + 4r_{10}\omega_{10}\varphi_{10}\cos\theta_{10} - 6r_{10}\omega_{10}^2\alpha_{10}\sin\theta_{10} + \\ +3r_{10}\alpha_{10}^2\cos\theta_{10} - r_{10}\omega_{10}^4\cos\theta_{10} \\ \\ r_{10}\psi_{10}\cos\theta_{10} + 4r_{10}\omega_{10}\varphi_{10}\sin\theta_{10} + 6r_{10}\omega_{10}^2\alpha_{10}\cos\theta_{10} + \\ +3r_{10}\alpha_{10}^2\sin\theta_{10} - r_{10}\omega_{10}^4\sin\theta_{10} \end{bmatrix} \tag{87}$$

where

$$\begin{aligned} A_7 = S_{Dx} - 3r_9\alpha_9^2\cos\theta_9 - 3r_{10}\alpha_{10}^2\cos\theta_{10} + r_9\omega_9^4\cos\theta_9 + \\ + r_{10}\omega_{10}^4\cos\theta_{10} + 6r_9\omega_9^2\alpha_9\sin\theta_9 + 6r_{10}\omega_{10}^2\alpha_{10}\sin\theta_{10} + \\ -4r_9\omega_9\psi_9\cos\theta_9 - 4r_{10}\omega_{10}\psi_{10}\cos\theta_{10} \end{aligned} \tag{88}$$

$$\begin{aligned} B_7 = -3r_9\alpha_9^2\sin\theta_9 - 3r_{10}\alpha_{10}^2\sin\theta_{10} + r_9\omega_9^4\sin\theta_9 + \\ + r_{10}\omega_{10}^4\sin\theta_{10} - 6r_9\omega_9^2\alpha_9\cos\theta_9 - 6r_{10}\omega_{10}^2\alpha_{10}\cos\theta_{10} + \\ -4r_9\omega_9\psi_9\sin\theta_9 - 4r_{10}\omega_{10}\psi_{10}\sin\theta_{10} \end{aligned} \tag{89}$$

The angular crackle ϕ_9, ϕ_{10} and the crackle vector $\mathbf{C}_E$ of point E are given by

$$\phi_9 = \frac{A_8 + B_8\tan\theta_{10}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{90}$$

$$\phi_{10} = -\frac{B_8 + r_9\phi_9\cos\theta_9}{r_{10}\cos\theta_{10}} \tag{91}$$

$$\mathbf{C}_E = \begin{bmatrix} r_{10}\phi_{10}\sin\theta_{10} + 5r_{10}\omega_{10}\psi_{10}\cos\theta_{10} + \\ +10r_{10}\alpha_{10}\varphi_{10}\cos\theta_{10} - 10r_{10}\omega_{10}^2\varphi_{10}\sin\theta_{10} + \\ -15r_{10}\omega_{10}\alpha_{10}^2\sin\theta_{10} - 10r_{10}\omega_{10}^3\alpha_{10}\cos\theta_{10} + \\ +r_{10}\omega_{10}^5\sin\theta_{10} \\ \\ -r_{10}\phi_{10}\sin\theta_{10} + 5r_{10}\omega_{10}\psi_{10}\sin\theta_{10} + \\ +10r_{10}\alpha_{10}\varphi_{10}\sin\theta_{10} + 10r_{10}\omega_{10}^2\varphi_{10}\cos\theta_{10} + \\ +15r_{10}\omega_{10}\alpha_{10}^2\cos\theta_{10} - 10r_{10}\omega_{10}^3\alpha_{10}\sin\theta_{10} + \\ -r_{10}\omega_{10}^5\cos\theta_{10} \end{bmatrix} \tag{92}$$

where

$$\begin{aligned} A_8 = C_{Dx} - r_9\omega_9^5\sin\theta_9 - r_{10}\omega_{10}^5\sin\theta_{10} + \\ +10r_9\omega_9^3\alpha_9\cos\theta_9 + 10r_{10}\omega_{10}^3\alpha_{10}\cos\theta_{10} + \\ +15r_9\omega_9\alpha_9^2\sin\theta_9 + 15r_{10}\omega_{10}\alpha_{10}^2\sin\theta_{10} + \\ +10r_9\omega_9^2\varphi_9\sin\theta_9 + 10r_{10}\omega_{10}^2\varphi_{10}\sin\theta_{10} + \\ -10r_9\alpha_9\varphi_9\cos\theta_9 - 10r_{10}\alpha_{10}\varphi_{10}\cos\theta_{10} + \\ -5r_9\omega_9\psi_9\cos\theta_9 - 5r_{10}\omega_{10}\phi_{10}\cos\theta_{10} \end{aligned} \tag{93}$$

$$\begin{aligned} B_8 = r_9\omega_9^5\cos\theta_9 + r_{10}\omega_{10}^5\cos\theta_{10} + \\ +10r_9\omega_9^3\alpha_9\sin\theta_9 + 10r_{10}\omega_{10}^3\alpha_{10}\sin\theta_{10} + \\ -15r_9\omega_9\alpha_9^2\cos\theta_9 - 15r_{10}\omega_{10}\alpha_{10}^2\cos\theta_{10} + \\ -10r_9\omega_9^2\varphi_9\cos\theta_9 - 10r_{10}\omega_{10}^2\varphi_{10}\cos\theta_{10} + \\ -10r_9\alpha_9\varphi_9\sin\theta_9 - 10r_{10}\alpha_{10}\varphi_{10}\sin\theta_{10} + \\ -5r_9\omega_9\psi_9\sin\theta_9 - 5r_{10}\omega_{10}\phi_{10}\sin\theta_{10} \end{aligned} \tag{94}$$

Finally, the angular pop γ_9, γ_{10} and the pop vector $\mathbf{P}_E$ of point E are given by

$$\gamma_9 = \frac{A_9 + B_9\tan\theta_{10}}{r_9\left(\sin\theta_9 - \tan\theta_{10}\cos\theta_9\right)} \tag{95}$$

$$\gamma_{10} = -\frac{B_9 + r_9\gamma_9\cos\theta_9}{r_{10}\cos\theta_{10}} \tag{96}$$

$$\mathbf{P}_E = \begin{bmatrix} r_{10}\gamma_{10}\sin\theta_{10} - 15r_{10}\alpha_{10}^3\sin\theta_{10} + 10r_{10}\varphi_{10}^2\cos\theta_{10} + \\ + r_{10}\omega_{10}^6\cos\theta_{10} + 15r_{10}\alpha_{10}\omega_{10}^4\sin\theta_{10} + \\ + 20r_{10}\omega_{10}^3\varphi_{10}\cos\theta_{10} - 15r_{10}\omega_{10}^2\psi_{10}\sin\theta_{10} + \\ -45r_{10}\omega_{10}^2\alpha_{10}^2\cos\theta_{10} + 15r_{10}\alpha_{10}\psi_{10}\cos\theta_{10} + \\ +6r_{10}\omega_{10}\phi_{10}\cos\theta_{10} - 60r_{10}\omega_{10}\alpha_{10}\varphi_{10}\sin\theta_{10} \\ \\ -r_{10}\gamma_{10}\cos\theta_{10} + 15r_{10}\alpha_{10}^3\cos\theta_{10} + 10r_{10}\varphi_{10}^2\sin\theta_{10} + \\ + r_{10}\omega_{10}^6\sin\theta_{10} - 15r_{10}\alpha_{10}\omega_{10}^4\cos\theta_{10} + \\ -20r_{10}\omega_{10}^3\varphi_{10}\sin\theta_{10} + 15r_{10}\omega_{10}^2\psi_{10}\cos\theta_{10} + \\ -45r_{10}\omega_{10}^2\alpha_{10}^2\sin\theta_{10} + 15r_{10}\alpha_{10}\psi_{10}\sin\theta_{10} + \\ +6r_{10}\omega_{10}\phi_{10}\sin\theta_{10} + 60r_{10}\omega_{10}\alpha_{10}\varphi_{10}\cos\theta_{10} \end{bmatrix} \quad (97)$$

where

$$\begin{aligned} A_9 = {} & P_{Dx} + 15r_9\alpha_9^3\sin\theta_9 + 15r_{10}\alpha_{10}^3\sin\theta_{10} + \\ & -10r_9\varphi_9^2\cos\theta_9 - 10r_{10}\varphi_{10}^2\cos\theta_{10} - r_9\omega_9^6\cos\theta_9 + \\ & -r_{10}\omega_{10}^6\cos\theta_{10} - 15r_9\omega_9^4\alpha_9\sin\theta_9 - \\ & +15r_{10}\omega_{10}^4\alpha_{10}\sin\theta_{10} + 20r_9\omega_9^3\varphi_9\cos\theta_9 \\ & +20r_{10}\omega_{10}^3\varphi_{10}\cos\theta_{10} + 15r_9\omega_9^2\psi_9\sin\theta_9 + \\ & +15r_{10}\omega_{10}^2\psi_{10}\sin\theta_{10} + 45r_9\omega_9^2\alpha_9^2\cos\theta_9 + \\ & +45r_{10}\omega_{10}^2\alpha_{10}^2\cos\theta_{10} - 15r_9\alpha_9\psi_9\cos\theta_9 + \\ & -10r_{10}\alpha_{10}\psi_{10}\cos\theta_{10} - 6r_9\omega_9\phi_9\cos\theta_9 + \\ & -6r_{10}\omega_{10}\phi_{10}\cos\theta_{10} + 60r_9\omega_9\alpha_9\varphi_9\sin\theta_9 + \\ & +60r_{10}\omega_{10}\alpha_{10}\varphi_{10}\sin\theta_{10} \end{aligned} \quad (98)$$

$$\begin{aligned} B_9 = {} & -15r_9\alpha_9^3\cos\theta_9 - 15r_{10}\alpha_{10}^3\cos\theta_{10} + \\ & -10r_9\varphi_9^2\cos\theta_9 - 10r_{10}\varphi_{10}^2\cos\theta_{10} - r_9\omega_9^6\sin\theta_9 + \\ & -r_{10}\omega_{10}^6\sin\theta_{10} + 15r_9\omega_9^4\alpha_9\cos\theta_9 + \\ & +15r_{10}\omega_{10}^4\alpha_{10}\cos\theta_{10} + 20r_9\omega_9^3\varphi_9\sin\theta_9 \\ & +20r_{10}\omega_{10}^3\varphi_{10}\sin\theta_{10} - 15r_9\omega_9^2\psi_9\cos\theta_9 + \\ & -15r_{10}\omega_{10}^2\psi_{10}\cos\theta_{10} + 45r_9\omega_9^2\alpha_9^2\sin\theta_9 + \\ & +45r_{10}\omega_{10}^2\alpha_{10}^2\sin\theta_{10} - 15r_9\alpha_9\psi_9\sin\theta_9 + \\ & -10r_{10}\alpha_{10}\psi_{10}\sin\theta_{10} - 6r_9\omega_9\phi_9\sin\theta_9 + \\ & -6r_{10}\omega_{10}\phi_{10}\sin\theta_{10} + -60r_9\omega_9\alpha_9\varphi_9\cos\theta_9 + \\ & -60r_{10}\omega_{10}\alpha_{10}\varphi_{10}\cos\theta_{10} \end{aligned} \quad (99)$$

3. GRAPHICAL AND NUMERICAL RESULTS

The kinematic analysis of the proposed 10-bar long-dwell mechanism of Fig. 1 was developed according to the formulation described above, which has been implemented in a Matlab program and validated by means of several examples. In particular, Fig. 3 shows a simulation of a 10-bar long-dwell mechanism with the following geometric characteristics: r_1 =58.30 u, $r_2 = r_9$ = 10 u, r_3=40 u, $r_4 = r_5 = r_7$ = 30 u, r_{10} = 20 u, where u is the unit length. This example refers to the kinematic input data: ω_2 = 1 r/s, $\alpha_2 = \varphi_2 = \psi_2 = \phi_2 = \gamma_2 = 0$. Tabs. 1 and 2 summarize the kinematic characteristics for θ_2 = 0°, which means to have all sub-mechanisms at the dead-point configuration. In particular, Tab. 1 shows the punctual kinematic characteristics of *A*, *B*, *C*, *D* and *E*, while Tab. 2 shows the angular kinematic characteristics of links 2, 3, 4, 5, 7, 9 and 10, where the last is the output rocker.

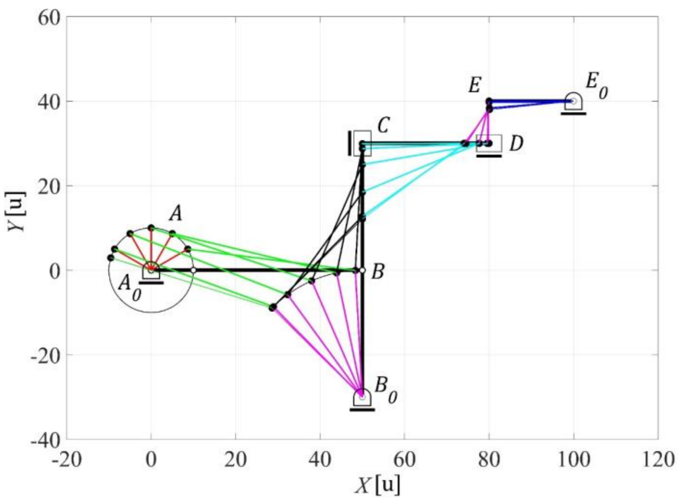

FIGURE 3: LONG-DWELL MECHANISM DURING MOTION.

TABLE 1: VELOCITY, ACCELERATION, JERK, SNAP, CRACKLE, POP OF POINTS *A*, *B*, *C*, *D* AND *E* WHEN θ_2 = 0°.

Points	**v** [u/s]	**a** [u/s^2]	**J** [u/s^3]	**S** [u/s^4]	**C** [u/s^5]	**P** [u/s^6]
A	10	10	10	10	10	10
B	0	12.5	0	32.9	14.1	184.2
C	0	0	0	-31.2	0	362.6
D	0	0	0	0	0	0
E	0	0	0	0	0	0

TABLE 2: VELOCITY, ACCELERATION, JERK, SNAP, CRACKLE, POP OF LINKS 2, 3, 4, 5, 7, 9 AND 10 WHEN θ_2 = 0°.

Links	ω [r/s]	α [r/s^2]	φ [r/s^3]	ψ [r/s^4]	ϕ [r/s^5]	γ [r/s^6]
2	1	0	0	0	0	0
3	0.2	0	0.23	-0.4	-0.1	4.2
4	0	0.4	0	-0.9	0.4	0
5	0	-0.4	0	0.9	0.4	0
7	0	0	0	1	0	-12.9
9	0	0	0	0	0	0
10	0	0	0	0	0	0

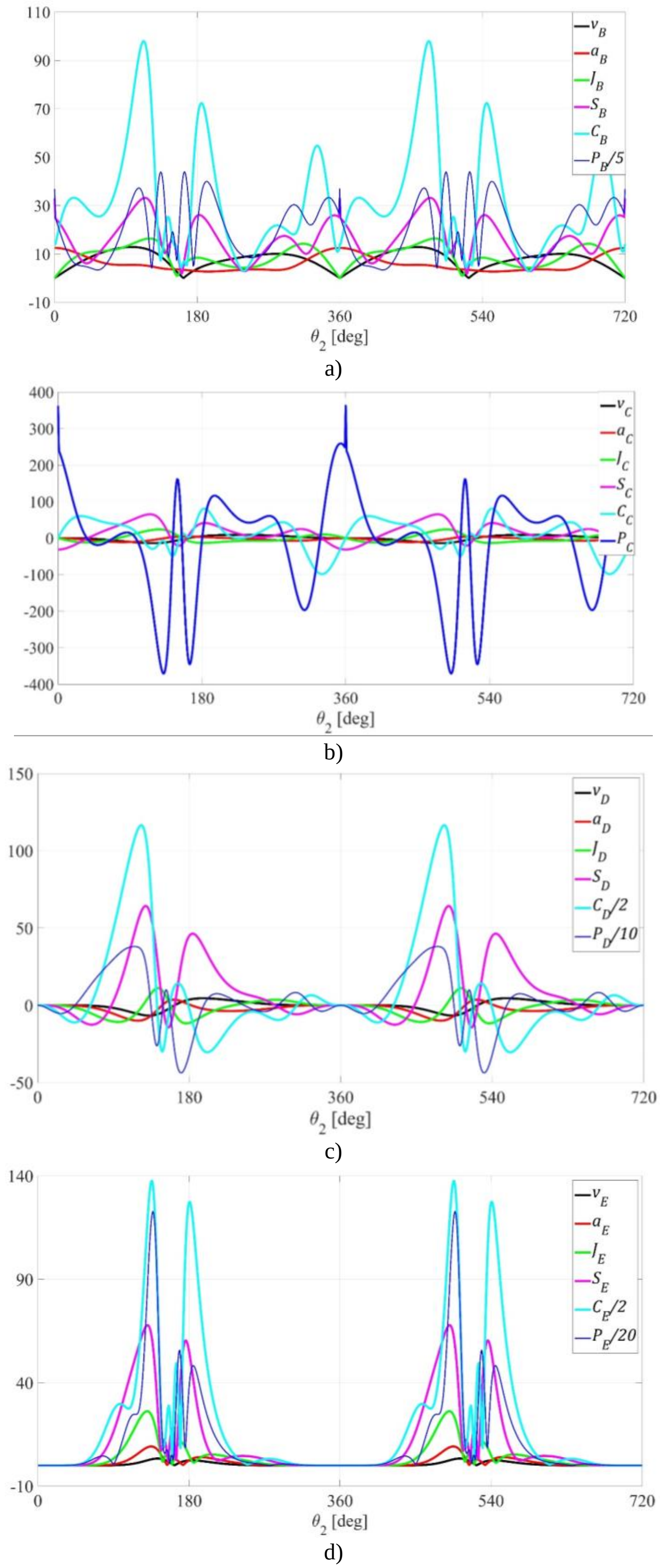

FIGURE 4: VELOCITY, ACCELERATION, JERK, JOUNCE OR SNAP, CRACKLE AND POP DIAGRAMS OF POINTS *B*, *C*, *D* AND *E* VERSUS THE CRANK ANGLE θ_2.

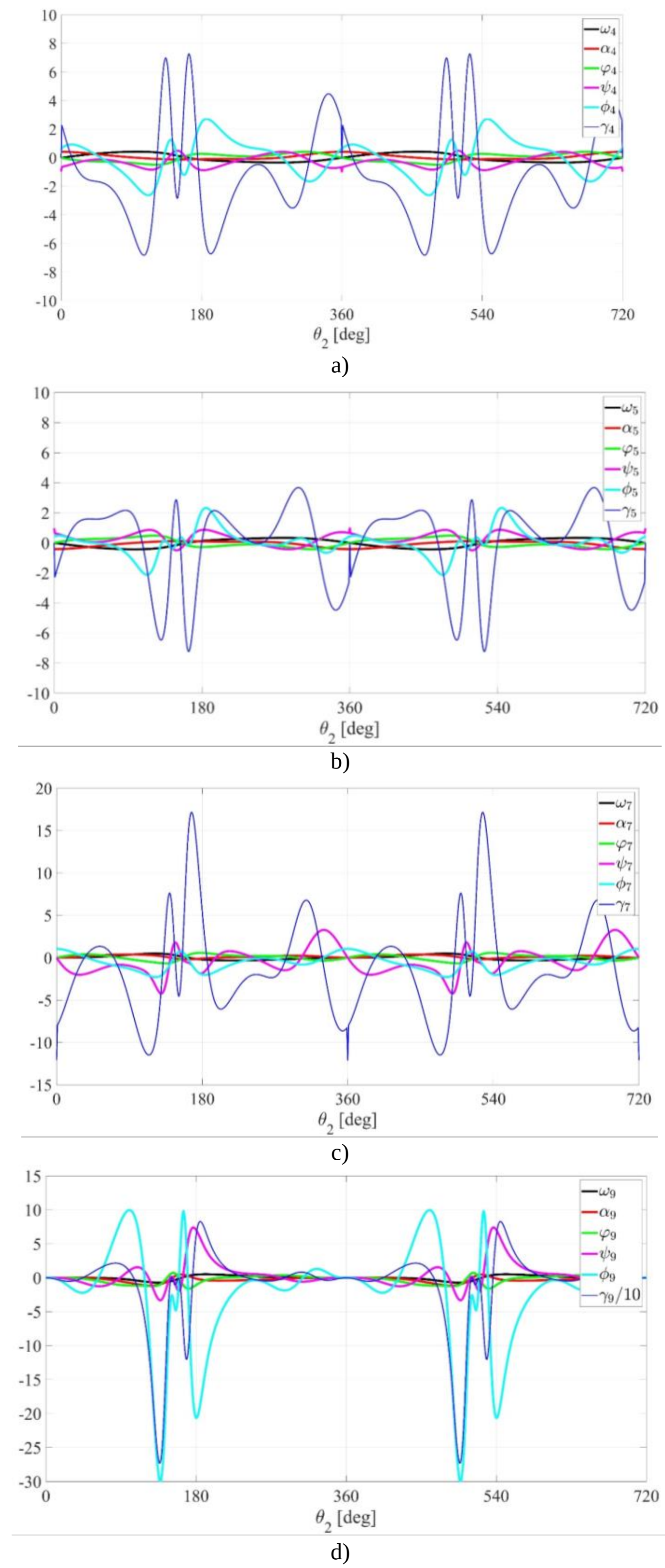

FIGURE 5: ANGULAR VELOCITY, ACCELERATION, JERK, JOUNCE OR SNAP, CRACKLE AND POP OF LINKS 4, 5, 7 AND 9 VERSUS THE CRANK ANGLE θ_2.

Moreover, Fig. 4 shows the diagrams of the punctual velocity, acceleration, jerk, jounce or snap, crackle and pop of points *B*, *C*, *D* and *E*, as function of the crank angle θ_2, while Fig.5 shows the diagrams of the angular velocity, acceleration, jerk, jounce or snap, crackle and pop of links 4, 5, 7 and 10, as function of the crank angle θ_2.

In particular, a very long-dwell is obtained on the output rocker link 10 by observing Figs. 4d and 5d in the angular range that is centered on 360°, which corresponds to the total dead-points configuration.

4. CONCLUSIONS

The higher order kinematic analysis of a 10-bar long-dwell mechanism that was synthesized by applying the dead-points superposition method and thus, connecting in series four mechanisms. This approach can be extended to the synthesis of *N*-bar long-dwell mechanisms with assigned dwell-time and kinematic characteristics, in terms of number and type of the assembled four-bar mechanisms with dead-points configurations, which can be of 4R, 3RP, 2R2P and RPRP types.

The proposed formulation was implemented in Matlab and validated by means of several graphical and numerical results.

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