ADVANCED CDM STRAIN-BASED MODELING FOR II+III CREEP UNDER MULTIAXIAL STATE OF STRESS

Nicola Bonora, Vittorio di Cocco and Domenico Gentile
DiMSAT – University of Cassino, Cassino, Italy 03043
nbonora@unicas.it

ABSTRACT

Advanced and reliable creep predicting modeling is needed for a large number of high temperature component design applications. Most of the models available in the literature, where different creep stages are approached separately, suffer from lacking of geometric transferability from specimen to component. In the last decades continuum damage mechanics (CDM) formulation for creep damage has been proposed. In the majority of models derived from this framework, creep damage evolution law, usually given as a function of the applied stress and time as in the form proposed initially by Kachanov, is commonly assumed. In this paper a different approach is proposed. Starting from the observation that creep damage, in the form of void nucleation and growth, is the result of the impossibility to no-longer redistribute at grain level the excess in deformation due to diffusive processes, creep damage evolution can be related to the accumulation of the inelastic strain. Since damage micro-mechanism for creep and plastic strain is the same, a single damage dissipation potential form for both phenomena can be adopted in the CDM formulation. Here, starting from the damage modeling proposed by Bonora (1997), a new strain-based CDM model for creep is developed and presented with particular emphasis to geometry, stress and temperature transferability predicting capabilities.

NOTATION AND SYMBOLS

D damage variable
A_{eff} effective area
A_0 nominal area
$\tilde{E}$ effective Young modulus
E initial Young modulus
ε^t total strain
ε^e elastic strain
ε^p plastic strain
ε^c creep strain
$\dot{\varepsilon}^{e}_{ij}$ elastic strain rate
ν Poisson ratio
$\dot{\sigma}_{ij}$ stress rate
δ_{ij} Kroneker delta
$\dot{\varepsilon}^{p}_{ij}$ plastic strain rate
$\dot{\lambda}$ plastic multiplier rate
f_p plastic dissipation potential
$\dot{s}_{ij}$ deviatoric stress tensor rate
σ_{eq} equivalent Mises stress
$\dot{\varepsilon}^{c}_{ij}$ creep strain tensor rate
T absolute temperature (°K)
A_1 thermal constant for creep law
n hadening exponent
Q_{att} activation energy
R isotropic hardening back stress
p effective active plastic strain (multiaxial)
r internal isotropic hardening variable
α damage exponent
ε_f theoretical uniaxial strain to failure
ε_{th} dmage threshold strain
σ_H/σ_{eq} triaxiality factor, TF
D_{cr} critical damage

INTRODUZION

The possibility to model and predict creep behavior of material and components under elevated temperature, and eventually severe stresses, is fundamental in a number of applications such as new generation turbine blades design. Creep strain accumulation is mainly due to diffusive processes that may occurs both in the material grains (diffusive creep) and along grain boundaries (dislocation climb) [2]. When inelastic strain cannot longer be redistributed along grain boundaries, microcavities nucleate resulting in the formation of macrocracks. Strain accumulation with time is a non linear process where usually three regions can be identified and commonly referred as primary (I) secondary (II) and tertiary (III) creep stage [3]. From the design point of view, many materials used in the applications show a very limited primary creep stage that is usually neglected. In the past a number of empirical relationships have been proposed in order to describe creep strain evolution with time such as the Bailey-Norton law for the secondary creep. The major limitation of these laws is that creep life cannot be predicted as well as the transition from secondary to tertiary creep, [4]. In addition, the identification of the effective creep response for a given stress level and temperature requires long and expensive testing since reliable short-duration testing techniques are not available. In this paper, alternatively to more traditional methodologies, where each creep phase is modeled separately, creep has been approached from a micromechanical point of view. Here, the appearance of tertiary creep stage has been interpreted as the resulting effect on material viscoplastic response due the activation and growth of damage mechanisms in the material microstructure. Starting from the continuum damage mechanics formulation initially developed by Kachanov [5], a novel creep-damage model is proposed. Here, creep damage is assumed to be time independent and function of the inelastic accumulated strain only. The proposed model is simple and requires a limited number of parameters that can be obtained from standard uniaxial tensile test. In this paper, the model formulation is presented and predicted creep lives for IMI837 titanium alloy, under both varying stress and temperature conditions, are compared with experimental data available in the literature.

CONSTITUTIVE MODELING

Under creep conditions, inelastic strain accumulates in the material microstructure in a time dependent manner. When the inelastic deformation cannot longer be accommodated in the microstructure, damage starts to occurs in the form of microvoids that can flow resulting, sooner or later, in the formation of macrocracks. Creep strain accumulation without damage usually take place during the entire secondary creep stage. When damage mechanisms are initiated, creep strain accumulation rate progressively increases due to local stress amplification caused by the reduction of the effective resisting section, figure 1.

According to this secondary creep stage can be understood as the effective pure viscoplastic (without damage) material response while tertiary creep can be interpreted as the viscoplastic material behavior modified by the presence of damage. From these considerations several continuum damage models have been proposed in the past always relating damage evolution to the imposed nominal stress, [6]-[7].

Creep and plastic deformations show a number of similarities. In both cases final rupture is always controlled by the nucleation and growth of voids that for plasticity are nucleated at the inclusions while in creep are nucleated preferentially at the grain boundaries and triple points. Since the microscopic damage mechanism is practically the same, it is possible to postulate that a dissipation potential for damage in creep exist and coincides with the one for plasticity damage. Consequently, creep damage is a time independent process that has to depend on total accumulated inelastic strain only. Time dependency is given by the vicoplastic material response that at high temperatures becomes more evident. Furthermore, there should be no substantial difference between damage effects induced by plasticity and creep at elevated temperatures. Here, the exceeding of material yield strength would only have the effect to accelerate damage development due to the superposition of plastic and creep strain accumulation.

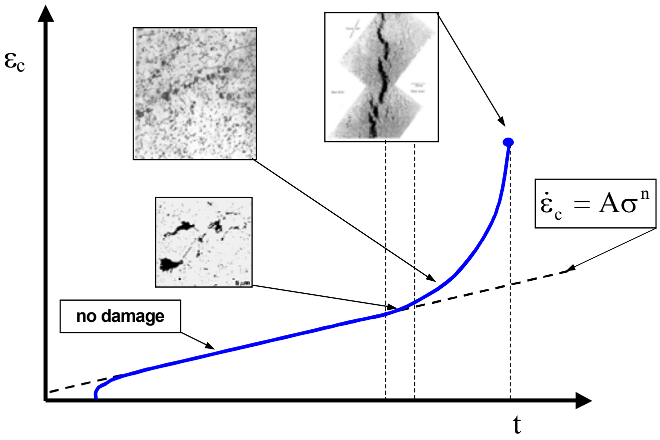


Figure. 1 – Damage evolution during creep strain accumulation.

Starting from these considerations the nonlinear damage model proposed by Bonora [1] and successively extended by Pirondi and Bonora [9] for complex load paths, has been assumed to be feasible to describe damage evolution as a function of inelastic strain that may accumulate as a consequence of both plasticity and creep. In the following the fundamental constitutive equations are summarized. Damage is defined as for plasticity:

$$D = 1 - \frac{A_{eff}}{A_0} = 1 - \frac{\tilde{E}}{E_0} \tag{1}$$

while total strain decomposition is assumed:

$$\varepsilon^t = \varepsilon^e + \left(\varepsilon^p + \varepsilon^c + \ldots\right) \tag{2}$$

where elastic deformation are given by,

$$\varepsilon_{ij}^e = \frac{1+\nu}{E}\frac{\sigma_{ij}}{1-D} - \frac{\nu}{E}\frac{\sigma_{kk}}{1-D}\delta_{ij} \tag{3}$$

Plastic flow is not affected by dmage and is given by

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda}\frac{\partial f_p}{\partial \sigma_{ij}} = \dot{\lambda}\frac{3}{2}\frac{\dot{s}_{ij}}{\sigma_{eq}} \tag{4}$$

Creep flow rate is given by,

$$\dot{\varepsilon}_{ij}^{c} = \dot{\varepsilon}_{c}\left(\sigma_{eq}, T; A_1, n, Q_{att}\right)\left(\frac{\partial \sigma_{eq}}{\partial \sigma_{ij}}\right) \tag{5}$$

where plastic multiplier is

$$\dot{r} = -\dot{\lambda}\frac{\partial f_p}{\partial R} = \dot{\lambda} = \dot{p}(1-D) \tag{6}$$

while the kinetic damage evolution law is given as

$$\dot{D} = -\dot{\lambda}\frac{\partial f_D}{\partial Y} = \alpha \cdot \frac{\left(D_{cr} - D_0\right)^{\frac{1}{\alpha}}}{\ln(\varepsilon_f / \varepsilon_{th})} \cdot f\left(\frac{\sigma_H}{\sigma_{eq}}\right) \cdot \left(D_{cr} - D\right)^{\frac{\alpha-1}{\alpha}} \cdot \frac{\dot{p}}{p} \tag{7}$$

In order to completely define the creep evolution law, the Norton expression is used replacing the actual stress with the effective one.

$$\dot{\varepsilon}_c = A\frac{\sigma}{1-D}^{\,n} \tag{8}$$

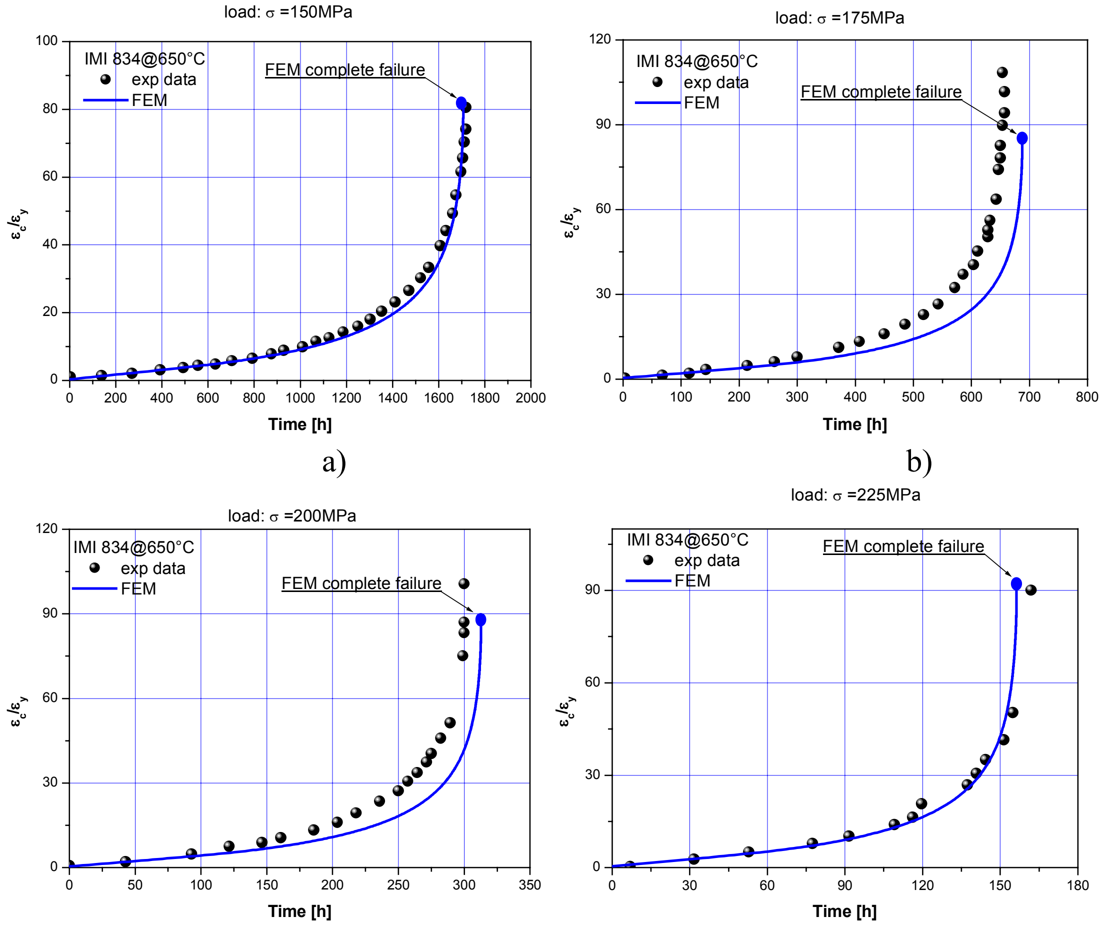
Here, temperature effect is also accounted in the constant A that can be expressed as a function of the activation energy as given in [10].

$$A(T) = A_1 \exp\left(\frac{Q_{att}}{KT}\right) \tag{9}$$

As formulated, the model requires four damage and three creep parameters. Even though the determination of the damage parameters can easily done at room temperature according to the procedure indicated in [1] practical difficulties can arise at elevated temperatures. Preliminary numerical studies have shown that the most critical parameter is the material damage strain threshold at which transition between secondary and tertiary creep occurs. At this stage of the research material and damage parameters have been calibrated according to a numerical-experimental procedure for a reference stress level and temperature. Once parameters were determined, they have been used to evaluate creep curve and time at rupture prediction at different stresses and temperatures.

CREEP CURVES AND TIME AT RUPTURE PREDICTIONS

The material under investigation is a Titanium alloy IMI834 tested at different stress levels and temperatures. Experimental data have been retrieved from the literature [11], for a temperature of 650°C and applied stress ranging from 150 to 225 MPa and for a stress level of 200 MPa and a temperature varying between 600 and 690°C, [12]. In addition, rupture time for round notch bar have been also used in order to verify the model prediction transferability from uniaxial to multiaxial state of stress conditions. According to the proposed model, failure occurs in the material when the uniaxial strain to failure is attained and damage reaches the critical value. Strain to failure is affected by stress triaxiality with consequent effect on time at rupture. The presence of necking in round bar specimen under constant stress would result in a reduction of material ductility and acceleration of creep strain accumulation. The proposed model has been implemented in commercial finite element code MSC MARC through a user subroutine and it has been used to predict creep curve and time at rupture. In figure 2a-d the comparison for T=600°C are given at different stress level as indicated in the captions.



a)

b)

c)

d)

Fig. 2 – Comparison between predicted creep curves and experimental data at 600°C for: a) σ=150MPa, b) σ=175MPa, c) σ=200MPa, d) σ=225MPa constant stress

IMI834 Material and
Norton law parameters

E=90000MPa
ν=0.3
σ_y=450MPa
A=4.60E-18
n=5.911

CDM damage parameters

ε_f=1.0
ε_{th}=0.02
D_{cr}=0.85
D_0=0.0
α=0.53

Table 1 – Material and damage parameters for IMI834.

It has to be noted that final failure is not imposed externally but it is the result of the simulation performed over an axisymmetric fem model of typical creep tensile specimen geometry. Here, creep strain is given normalized with respect to the nominal strain at first yield. Material and damage parameters used in the simulations are given in Table 1.

Comparison given in figures 2b-2d shows a very good agreement between the predicted creep strain accumulation with time together with both time and creep strain at rupture. These results confirm the transferability of damage parameter, identified for a specific stress level and temperature, with respect to other stress levels. The same damage parameters have been used to verify the model prediction transferability with respect to temperature. It is known that temperature has a strong effect on material ductily. At elevated temperatures, small temperature changes can results in large variation of material strain to failure. Since two out four damage parameters are strains, it has been proposed to updated material strain to failure at different temperatures using Johnson and Cook type law:

$$\varepsilon_f = \varepsilon_f^*(1 + D_5 T^*) \qquad (10)$$

where T^* is the homologous temperature, ε_f is the reference material strain to failure and D_5 is a material parameter. Speculations about temperature effect on material damage threshold are less immediate. In this case probably is not completely incorrect to postulate that increasing temperature would facilitate the diffusion of cavities along the grain boundary with consequent early development of damage and a reduction of the secondary creep stage. Consequently material damage threshold should decrease with temperature rise. In figure 3, the preliminary result relative to the prediction of creep accumulation curve with time for the same material under investigation at the temperature of 690°C is given confirming that the model has the potential to be robustly extended to properly account for both stress and temperature variation.

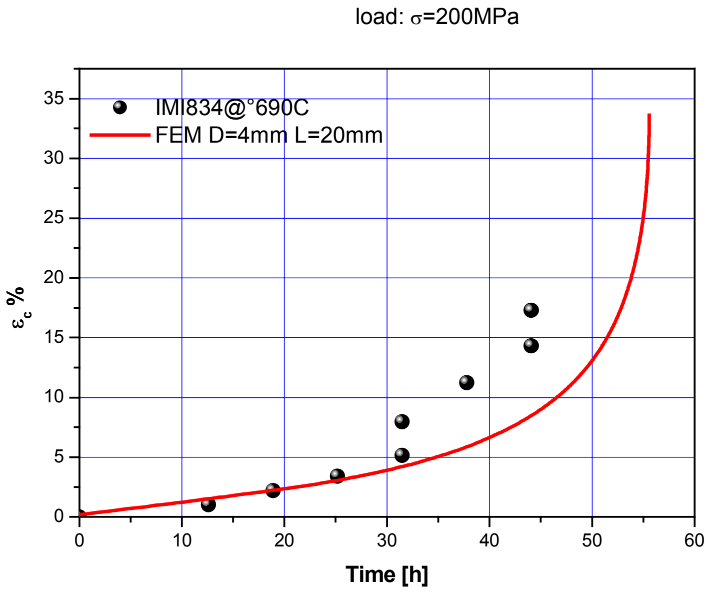


Fig. 2 – Predicted creep life at 690°C

Finally, model prediction transferability with respect to stress triaxiality has been also verified comparing the predicted time at rupture in round notched bar. Here, a reference configuration with a nominal stress concentration factor of K_t=1.4 (R=2.5 mm) has been simulated with FEM. A parametric study varying the imposed remote stress and determining the correspondent life at rupture has been performed. Standard Mises criterion for multiaxial creep has been assumed. Time at rupture has been compared with data available in the literature for the material under investigation at the reference temperature of 650°C at which damage parameters were previously determined, figure 3.

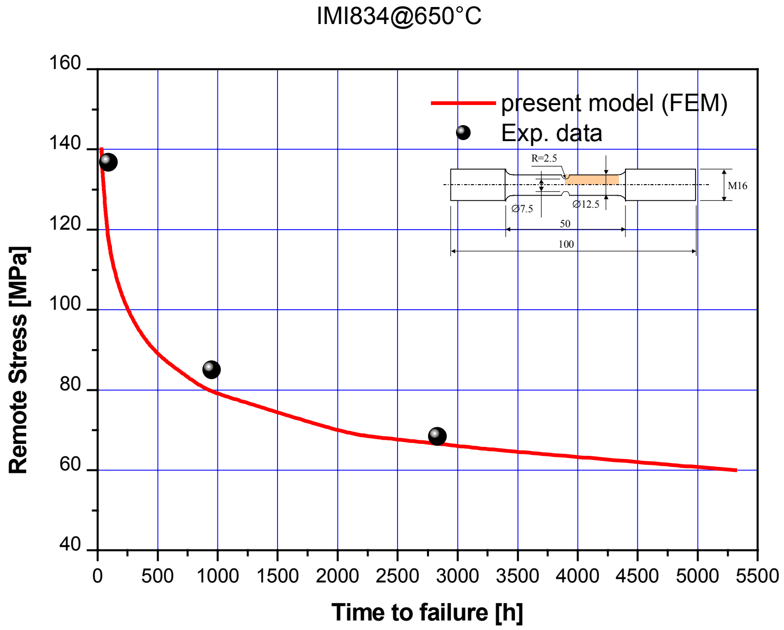


Fig. 3 – Creep time at rupture in round notch bar as a function of applied remote stress.

CONCLUSION

In this paper, starting from traditional continuum damage mechanics approach, a new strain based damage model for creep has been proposed. Using the similarities between damage induced by plastic strain and creep, creep damage has been related to the total accumulated inelastic strain. This assumption lead to a time independent formulation for creep damage accumulation. The proposed model has been used to predict creep life at different stress level, temperatures and stress triaxiality loading conditions. Preliminary comparisons with experimental data available in the literature seem to confirm the potential of the proposed formulation particularly with respect to transferability from specimen to components.

REFERENCES

[1] Bonora N., “A nonlinear CDM model for ductile fracture”, *Engineering Fracture Mechanics,* Vol. 58, no.1/2, 1997, pp. 11-28.

[2] S. Kerry, M. R. Winstone, “Creep behaviour of Ti_3Al-based titanioum aliminide alloys containing molybdenum”, *Materials Science and Engeneering*, A192/193, 1995, pp. 856-861.

[3] Lemaitre J., Chaboche J.-L., “Mechanics of solid materials”, Cambridge university press, Greath Britain, 1990.

[4] Oding I. A., Ivanova V. S., Burdukskii V. V., Geminov V. N., “Creep and stress relaxation in metals”, Oliver and Boyd, Edinburgh and London, 1966.

[5] Kachanov L. M., “Introduction to continum damage mechanics”, Martinus Nijhoff Publishers, Netherlands, 1986.

[6] J. M. Jenkins, “Comparison of Measured Temperaturees, Thermal Stresses and Creep Residues With Predictions on Built-up Titanium Structure“, *NASA Technical Memorandum 86814*, November 1987.

[7] W. Qi, A. Bertram, “Anisotropic continuum damage modeling for single cristals at high temperatures”, *International Journal of Plasticity*, 15, 1999, pp. 1197-1215.

[8] V. Di Cocco, D. Gentile e N. Bonora, “Un modello di danno per la previsione del cedimento per creep nei metalli”, *Proc. IGF XVI, Catania*, 2002, pp. 357-365 .

[9] A. Pirondi, N. Bonora, “Modeling Ductile Damage Under Fully Reversed Cycling”, *to appear in Computational Solids Mechanics*, 2002.

[10] Findley W. N., Lai J. S., ONARAN K., “Creep and relaxation of nonlinear viscoplastic materials”, Dover Publications, Inc., New York, 1989.

[11] Hyde T. H., Becker A. A., “Prediction of creep failure in aeroengine materials under multi-axial stress states”, *Int. J. Mech. Sci.*, Vol. 38, No.4, Great Britain, 1996, pp. 385-403.

[12] D. J. Smith, “Simulating the cyclic mechanical response of titanium alloy 834 at 630°C”, *Matherials at high temperatures*, 18(3), 2001, pp. 153-162.

[13] Min Lu, J. Hemker, “Intermediate temperature creep properties of gamma TiAl”, *Acta Mater.*, Vol. 45, N° 9, 1997, pp. 3573-3585.