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DYNAMIC MODEL OF A 3T1R PARALLEL KINEMATICS MACHINE

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ABSTRACT

This paper presents the dynamic model of a parallel kinematic manipulator (PKM) able to generate the Schönflies motion of its moving platform. The motion consists of a spatial translation (3T) with a rotation about a fixed axis (1R). The PKM consists of a fixed base connected to a moving platform by four kinematic chains with four prismatic actuators at the base. The model was developed in the framework of the Lagrange formulation according to a multibody approach with an augmented formulation without the need of solving the kinematics in terms of the independent coordinates. The model has been used to solve both the inverse and forward problems. In the former the trajectory of the moving platform is treated as a servo-constraint reducing the PKM as a kinematically driven mechanism, in the latter the constraint equations at acceleration level are adjoined to the equations of motion leading to a differential-algebraic equations. Finally, the dynamic model was validated via a solid model simulation.

Keywords: Computational dynamics; Schönflies motion; parallel kinematic machine; Lagrange formulation.

1 INTRODUCTION

The focus of this work is on a special class of parallel manipulators able to produce the Schönflies motion of the moving platform. This motion is a subgroup of the Euclidean group $SE(3)$ which allows for three independent translations and a rotation about a fixed axis. Because of the nature of the motion the manipulator is indicated as 3T1R, too. Various designs have been proposed either with serial or parallel architectures. The former can boast a large workspace and a long reach, the latter high load-carrying capacity, speed, stiffness and lightweight architecture making them ideal for pick-and-place operations. The first Schönflies motion generator (SMG), with serial architecture, was the Selective-Compliance-Assembly-Robot-Arm (SCARA) [1]. On the other hand, most of the parallel manipulators proposed came from the idea reported in [2] then developed by the same research group in [3–5]. These ideas led to commercialized robots as Adept Quattro and the Veloce. There are, indeed, numerous works in literature on 3T1R robots inspired by the cited works [6–10]. Another very common SMG design comes from the Delta-based architectures [11]. For example, IRB 340 Flexpicker, manufactured by ABB Robotics, is a Delta-based robot provided by a telescopic Cardan shaft to actuate the rotational motion. Besides, there are some researchers who have proposed a two-limbs SMG design in order to reduce the complexity and the cost of the robot [12, 13]. Great attention was paid to design the SMG for enhancing the rotational capability of the end-effector. In Quattro and its developments, for example, there was the relative movement between two sub-platforms amplified by various transmission systems as rack-pignon, gears [14] ans screw mechanisms [3, 15]. Despite of the considerable work on the kinematics and design of the SMG, there is definitely less work dedicated to their dynamics. In general, Newton-Euler method [16–18], method based on the principle of virtual work [19] and the Lagrangian formulation [20–22] are invariably used to establish the relationship between actuated torques/forces and the motion of the robot. The Newton-Euler method needs the equations of motion for each body of the robot leading to a large number of equations

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with poor computational efficiency. Method based on the principle of virtual work represents a more efficient technique for obtaining the equations of motion although it still requires the analysis of the forces/torques on each body of the robot. As it is well known, the Lagrangian formulation avoids to deal with bodies equilibrium as it introduces the calculation of the kinetic and potential energy of the manipulator. However, it is very demanding or even practically impossible to derive explicit equations of motion in terms of a set of independent coordinates because of the constrains imposed by the closed loops. Therefore, it can be convenient to write the equations of motion in terms of redundant coordinates adjoined to the constraint equations forming a system of differential-algebraic equations. In this paper, we build a dynamic model of a 4-PUU* manipulator belonging to a special class of Schönflies motion generators. Because of its geometry, a Lagrangian approach was used without introducing relevant simplifications and by means of redundant coordinates involving either the actuators or the moving platform kinematics. The paper extends the work presented in [23] where only the direct dynamics was solved. The motivation of this work is to present a general procedure to solve both the forward and inverse dynamic problems for this class of Schönflies motion generators.

The paper is organized as follows. Section 2 outlines the formulation used to solve the dynamic problems of the PK machines. Section 3 lists the steps for modeling the dynamic model of a PKM. Sections 2, 3 are general and therefore valid for any type of PKM. Section 4 introduces the geometry of the manipulator under study while section 5 shows the simulation results. Section 6 draws the conclusions of the work.

2 FORMULATION OF THE DYNAMIC PROBLEMS FOR PKM

Equations of motion of a PKM can be written in terms of n redundant coordinates $\mathbf{q}(t)$ subjected to m holonomic kinematic constraints $\mathbf{C}(\mathbf{q}(t)) = \mathbf{0}$ derived from its geometry:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{C}_q^T \boldsymbol{\lambda} = \mathbf{f}. \tag{1}$$

In Eq. (1), $\mathbf{M} \in \mathbb{R}^{n\times n}$, is the generalized inertia matrix, $\mathbf{V} \in \mathbb{R}^n$, is the Coriolis/centrifugal forces vector, $\mathbf{G} \in \mathbb{R}^n$, is the vector of gravitational forces and $\mathbf{f} \in \mathbb{R}^n$, is the vector of forces applied to the system including the motors torques/forces. Term $-\mathbf{C}_q^T\boldsymbol{\lambda}$ represents the constraints forces with $\mathbf{C}_q \in \mathbb{R}^{n\times m}$ being the Jacobian matrix of the constraints with respect to the coordinates, $\mathbf{C}_q = \partial\mathbf{C}/\partial\mathbf{q}$, and $\boldsymbol{\lambda} \in \mathbb{R}^m$, is the vector of the Lagrangian multipliers. We define the inverse and forward dynamical problems as follows.

*P stands for prismatic joint, U for universal joint.

2.1 INVERSE DYNAMICS

Given the motion, at least 2-times continuously differentiable, of the moving platform, $\boldsymbol{\gamma}(t)$, expressed by the coordinates $\mathbf{p} \in \mathbf{q}$ with $\boldsymbol{\gamma}(t) = \mathbf{p}$ and the external forces applied to the moving platform, $\mathbf{f}_e \in \mathbf{f}$, let calculate the forces/torques exerted by the motors $\boldsymbol{\tau} \in \mathbf{f}$. The imposed motion is treated as a servo-constraint such that the PKM becomes a kinematically driven mechanism whose inverse dynamic problem can be solved straightforwardly.

- Position problem
  The vector of the constraint equations is augmented by the servo-constraint and takes dimension n:

$$\mathbf{C}(\mathbf{q}(t), t) = [C_1(\mathbf{q}), \cdots, C_m(\mathbf{q}), \mathbf{p} - \boldsymbol{\gamma}(t)]^T, \tag{2}$$

  as well as the Jacobian matrix takes $n \times n$ dimension:

$$\mathbf{C}_q = \begin{bmatrix} \frac{\partial C_1}{\partial q_1} & \cdots & \frac{\partial C_1}{\partial \mathbf{p}} \\ \vdots & \ddots & \vdots \\ \frac{\partial C_m}{\partial q_1} & \cdots & \frac{\partial C_m}{\partial \mathbf{p}} \\ \mathbf{0}_{(n-m)\times m} & & \mathbf{1}_{(n-m)\times(n-m)} \end{bmatrix}. \tag{3}$$

  Further, $\mathbf{C}_q$ is a nonsingular matrix as the constraint equations are linearly independent since the PKM is assumed not to be overconstrained. At each instant the constraint equations can be solved numerically. For example, the iterative Newton algorithm leads to the updated position vector:

$$\mathbf{q}_{k+1} = \mathbf{q}_k - \mathbf{C}_{q_k}^{-1}\mathbf{C}(\mathbf{q}_k), \tag{4}$$

  with stopping condition: $\|\mathbf{C}_{q_k}^{-1}\mathbf{C}(\mathbf{q}_k)\| \leq \epsilon$.
  k is the iteration number and ϵ is the iteration threshold.
- Velocity problem
  A time derivative of the constraint vector allows to solve the problem at the velocity level:

$$\dot{\mathbf{q}} = -\mathbf{C}_q^{-1}\mathbf{C}_t \tag{5}$$

  with

$$\mathbf{C}_t = [\mathbf{0}_{1\times m}, \dot{\boldsymbol{\gamma}}]^T. \tag{6}$$

- Acceleration problem
  A time derivative of the velocity equation $\mathbf{C}_q\dot{\mathbf{q}} + \mathbf{C}_t = \mathbf{0}$, provides the acceleration equation:

$$(\mathbf{C}_q\dot{\mathbf{q}})_q\dot{\mathbf{q}} + \mathbf{C}_q\ddot{\mathbf{q}} + \mathbf{C}_{tt} = \mathbf{0}, \tag{7}$$

  with

$$\mathbf{C}_{tt} = [\mathbf{0}_{1\times m}, \ddot{\boldsymbol{\gamma}}]^T, \tag{8}$$

  such that

$$\ddot{\mathbf{q}} = -\mathbf{C}_q^{-1}(\mathbf{C}_{tt} + (\mathbf{C}_q\dot{\mathbf{q}})_q\dot{\mathbf{q}}). \tag{9}$$

Eqs. (4), (5) and (9) solve the PKM kinematics computing all the coordinates from the motion of the moving platform. This calculation can be regarded as a pre-computation of the inverse dynamics that can be solved managing eq. (1). In the inverse dynamics context, eq. (1) has n unknowns: m Lagrangian multipliers $\boldsymbol{\lambda}$ and $(n-m)$ actuators forces/torques $\boldsymbol{\tau}$. The unknowns are partially uncoupled, in fact it is convenient to select firstly the m equations that contain only the Lagrangian multipliers such that

$$\boldsymbol{\lambda} = \mathbf{C}_q^{\star -T}(\mathbf{f}_e - \mathbf{M}^\star \ddot{\mathbf{q}} - \mathbf{V}^\star - \mathbf{G}^\star). \tag{10}$$

Eqs. (10) is a set of linear equations whose solution is straightforward. Matrices $\mathbf{M}^\star$ and $\mathbf{C}_q^{\star T}$ are square with dimension $m \times m$, vectors $\mathbf{V}^\star$ and $\mathbf{G}^\star$ have m terms. Vector $\mathbf{f}_e$ contains m external forces, if any, that are known in the context of inverse dynamics. Once the Lagrangian multipliers are calculated the rest of equations, *i.e.* $(n-m)$, can be used to compute the actuators forces/torques such that

$$\boldsymbol{\tau} = \mathbf{M}^\dagger \ddot{\mathbf{q}} + \mathbf{V}^\dagger + \mathbf{G}^\dagger + \mathbf{C}_q^{\dagger T}\boldsymbol{\lambda}. \tag{11}$$

In eqs. (11), $\mathbf{M}^\dagger$ is $(n-m) \times n$, $\mathbf{C}_q^{\dagger T}$ is $(n-m) \times m$, vectors $\mathbf{V}^\dagger$ and $\mathbf{G}^\dagger$ have $(n-m)$ terms.

2.2 FORWARD DYNAMICS

Given the forces applied to the moving platform $\mathbf{f}_e \in \mathbf{f}$ and the forces/torques exerted by the motors $\boldsymbol{\tau} \in \mathbf{f}$, let calculate the motion of the moving platform $\mathbf{p} \in \mathbf{q}$. In the context of the forward dynamics the n equations of motion eq. (1) has $n+m$ unknowns, that is, $\boldsymbol{\lambda}$ and $\ddot{\mathbf{q}}$. To solve, we follow the augmented formulation in which the constraint equations at acceleration level are adjoined to eqs. (1) forming a set of differential-algebraic equations (DAE).

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_q^T \\ \mathbf{C}_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} - \mathbf{V} - \mathbf{G} \\ -(\mathbf{C}_q \dot{\mathbf{q}})_q \dot{\mathbf{q}} \end{bmatrix}. \tag{12}$$

Eq. (12) can be solved directly as the leading matrix is non singular. Alternatively, $\boldsymbol{\lambda}$ can be eliminated by exploiting the orthogonality between the constraint forces and any admissible $\dot{\mathbf{q}}$ which belongs to the null space of $\mathbf{C}_q$ since that $\mathbf{C}_q\dot{\mathbf{q}} = \mathbf{0}$. Thus, the first matrix block of Eq. (12) is projected onto the transpose of the null space of $\mathbf{C}_q$ (orthogonal complement of $\mathbf{C}_q$), namely $\mathbf{B}^T$, obtaining the following system:

$$\begin{bmatrix} \mathbf{B}^T\mathbf{M} \\ \mathbf{C}_q \end{bmatrix} \ddot{\mathbf{q}} = \begin{bmatrix} \mathbf{B}^T(\mathbf{f} - \mathbf{V} - \mathbf{G}) \\ -(\mathbf{C}_q \dot{\mathbf{q}})_q \dot{\mathbf{q}} \end{bmatrix}. \tag{13}$$

Eqs. (13) is a set of n ordinary differential equations as $\mathbf{B}^T \in \mathbb{R}^{(n-m)\times n}$. Matrix $\mathbf{B}$ can be computed either via QR, eigen or singular value decompositions of $\mathbf{C}_q$. Any integration method can, then, be used to obtain $\dot{\mathbf{q}}(t)$ and $\mathbf{q}(t)$ via a standard fixed/variable step size explicit method starting from known initial configuration and velocity.

3 PKM MODEL PROCEDURE

In order to model a PKM composed by $i = 1, \cdots, n_l$ legs, a fixed base and a moving platform mp the following general procedure is outlined:

- Definition of the Denavit-Hartenberg (DH) homogeneous transformation matrices for leg $i^{th\cdot}$;
- Definition of the redundant coordinates;
- Definition of the kinematic constraints;
- Analytical calculation of the terms of eq. (1);
- Numerical implementation.

To model the kinematics of the legs is mandatory to define the coordinates of the PKM to which the moving platform coordinates will be added to. It will turn out a redundant number of coordinates constrained by the vector-loop equations connecting the base to the moving platform by means of the legs kinematic chains. Therefore, terms of eq. (1) can be obtained by a simple analytical computation of the kinetic $K = K_{mp} + \sum_1^{n_l} K_i$ and gravitational $U = U_{mp} + \sum_1^{n_l} U_i$ energy of PKM according to Lagrange formulation:

$$\mathcal{L} = K - U;$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}). \tag{14}$$

As well as, the vector of the forces $\mathbf{f}$ is obtained directly from the virtual work with respect to the redundant coordinates without the need of introducing the Jacobian matrix, that maps the dependent coordinates into the independent ones.

4 PKM GEOMETRY

Figure 1 shows the PKM under study. The manipulator consists of a fixed base and a moving platform connected by four legs. Each leg is a serial kinematic chain with an actuated prismatic joint (P), a two couple of revolute joints (R) with their axes being perpendicular and coplanar, namely a couple of R joints forms a U joint. For each leg, the axis of the P joint is normal to the base and parallel to the first and last R joints of the leg. The second and third R joint axes are parallel, too. The motion of the moving platform is a spatial translation with a rotation about a fixed axis known as Schönflies motion. This 3T1R PKM was synthesized by X. Kong and C. Gosselin [24] whereas the complete kinematic analysis was solved by M. Ruggiu [25]. In order to model the PKM we define the reference systems on the $i^{th\cdot}$ leg according to the standard DH convention and the reference systems either on the base or on the moving platform, respectively, $\mathcal{G} : \{O_G XYZ\}$, $\mathcal{P} : \{PUVW\}$. Figure 2 shows the reference systems on the PKM, Table I shows the DH parameters for the leg†.

It is worth noting that only three coordinates (joints variables) can represent the leg kinematics to be included

†For sake of clarity the index i for the leg is omitted in Figure 2.

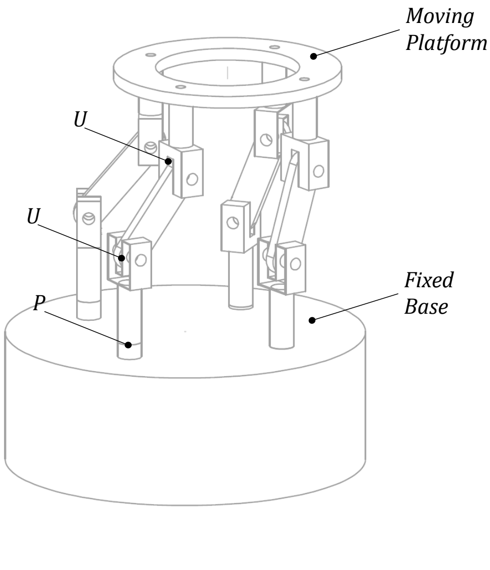

Figure 1 The Schönflies motion generator.

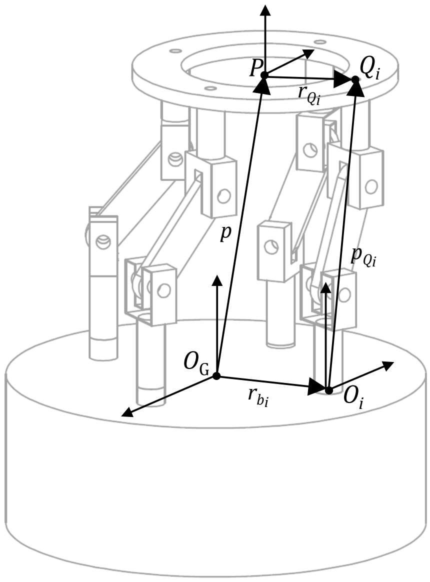

Figure 3 Vector kinematic chain loop.

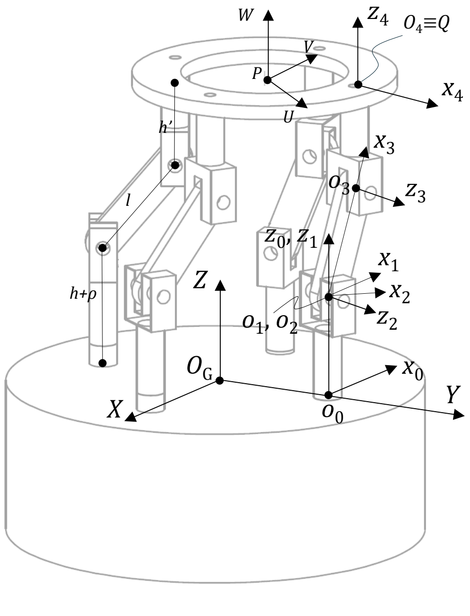

Figure 2 Reference systems for the PKM.

Table I - DH-table, leg i

link/joint j	α_j	a_j	d_j	$\theta_{j,i}$
1	0	0	$\rho_i + h$	0
2	$\frac{\pi}{2}$	0	0	$\theta_{1,i}$
3	0	l	0	$\theta_{2,i}$
4	$-\frac{\pi}{2}$	0	h'	$-\theta_{2,i}$

into dynamic model to which the coordinates defining the pose of the moving platform are added. In total, we deal with 16 coordinates, $\mathbf{q} = \left[\{\rho_i, \theta_{1i}, \theta_{2i}\}_1^4, x, y, z, \phi\right]^T$. There are, however, the vector loop equations representing the kinematic constraints of the PKM where the coordinates are related. Thus, summing up, there are only four independent coordinates, as expected.

4.1 KINEMATIC CONSTRAINTS

As it is said, the constraint equations $\mathbf{C}(\mathbf{q}) = \mathbf{0}$ are the loop equations from point O_G to P following each kinematic chain of the leg (Figure 3):

$$^G\mathbf{r}_{b_i} + {}^{0_i}\mathbf{R}_G^T\,{}^{0_i}\mathbf{p}_{Q_i} - {}^G\mathbf{p} - {}^G\mathbf{Q}_P\,{}^P\mathbf{r}_{Q_i} = \mathbf{0} \qquad (15)$$

where ${}^{0_i}\mathbf{p}_{Q_i}$ is expressed in the base reference system of the leg i and it contains the leg joint variables, ${}^G\mathbf{p}$ provides the position of the moving platform centre of mass, x, y, z, ${}^G\mathbf{Q}_P$ is the matrix that represents the orientation of the moving platform. The moving platform can only rotate by ϕ about z_G such that ${}^G\mathbf{Q}_P \equiv {}^G\mathbf{R}_z(\phi)$. Vectors in eq. (15) are expressed in the PKM base reference system, matrices ${}^{0_i}\mathbf{R}_G^T$ and vectors ${}^G\mathbf{r}_{b_i}$, ${}^P\mathbf{r}_{Q_i}$ are constants whose values depend of which leg is considered. Thus, there are $m = 12$ constraint equations to obtain $f = n - m = 4$ degrees of freedom for the PKM, as expected. The (12×16) Jacobian constraint matrix $\mathbf{C}_q$ can, thus, be easily computed.

5 SIMULATION RESULTS

The correctness and the the reliability of the model proposed was verified by two different tests.

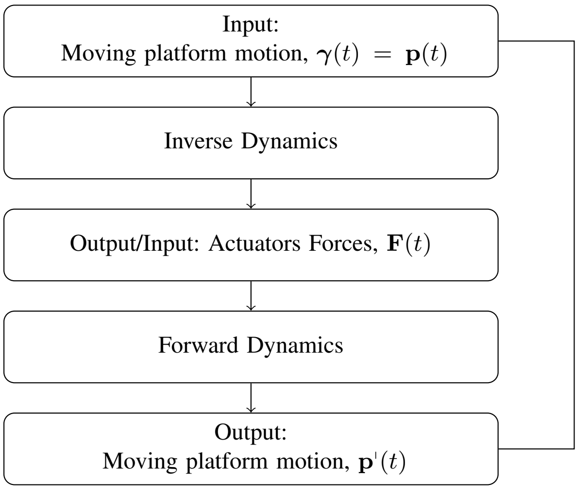

Figure 4 Auto-Test routine.

Table II - PKM geometrical parameters

h (mm)	l (mm)	h' (mm)	r_b (mm)	r_Q (mm)
66.34	80	75	92.73	75

5.1 AUTO-TEST

The correctness of the model was tested by an *auto-test* routine shown in Figure 4.

Either the inverse or the forward codes will be auto-proved whenever $\mathbf{p}(t) \equiv \mathbf{p}'(t)$. The simulation consists of a $2s$ long prescribed motion of the moving platform described by a sixth-order polynomial for each of the coordinates. The motion is free from both kinematic and constraint singularities such that the actuators forces are bounded and the constraint Jacobian is invertible. All geometric parameters and mass properties of the PKM used for the simulation are given in Tables II and III, respectively.

Link 1 only translates, all the bodies are considered homogeneous. Figure 5 shows the driven forces obtained from the inverse dynamics and then used to solve the forward dynamics. The integration method used was the fourth order Runge-Kutta with step size of 0.001s. Figure 6 shows the prescribed motion and the motion computed by the forward dynamics code. They practically coincide each to the other.

The validation was carried out through the forward dynamics computation. The simulation consists of a 1.5s motion with motors 2 and 3 not activated and the gravity effect omitted. The prescribed motors laws are given in Figure 8.

Table III - PKM mass parameters

	mass (g)	inertia $(kgmm^2)$
link 1	93.31	–
link 3	63.71	diag$(3.83, 49.86, 52.64)$
moving platform	454.74	diag$(0, 0, 2874.21)$

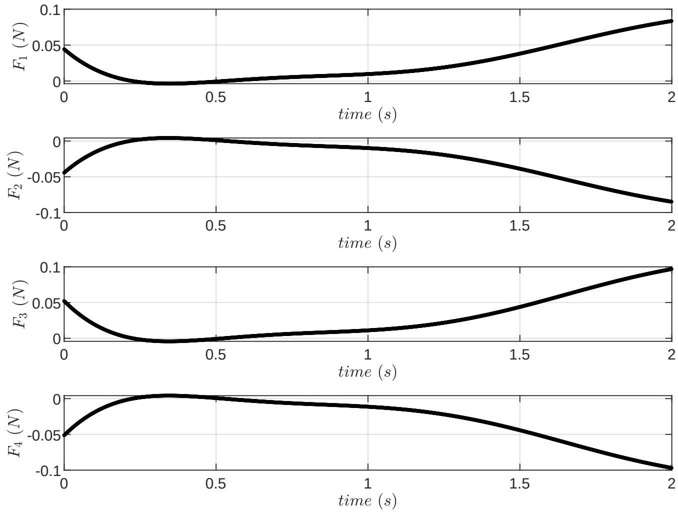

Figure 5 Actuators Forces of the *Auto-Test* simulation.

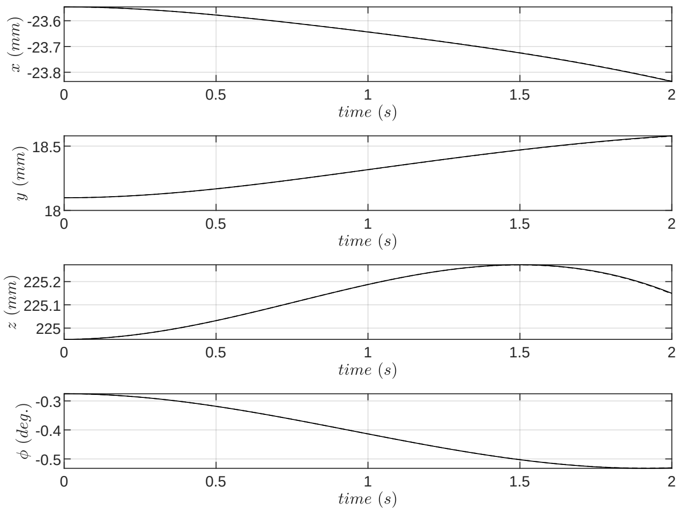

Figure 6 *Auto-Test* results -: computed, –: prescribed.

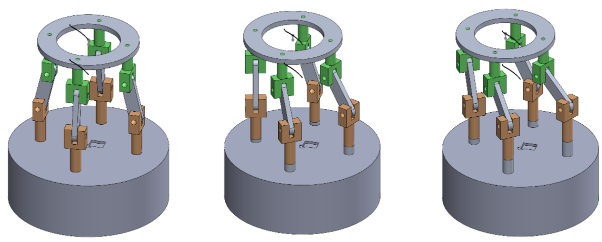

Figure 7 PKM solid model motion frames.

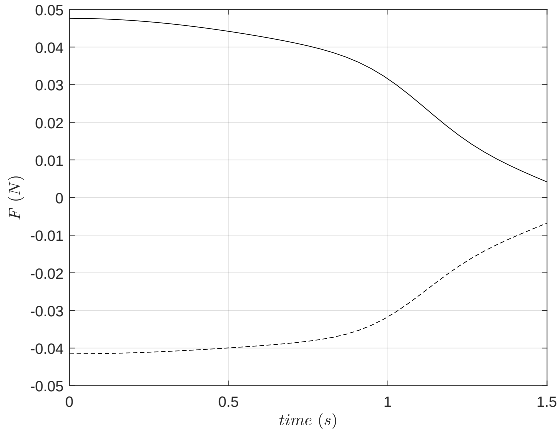

Figure 8 Motor forces of the comparison test:
F_1(—), F_4(- -).

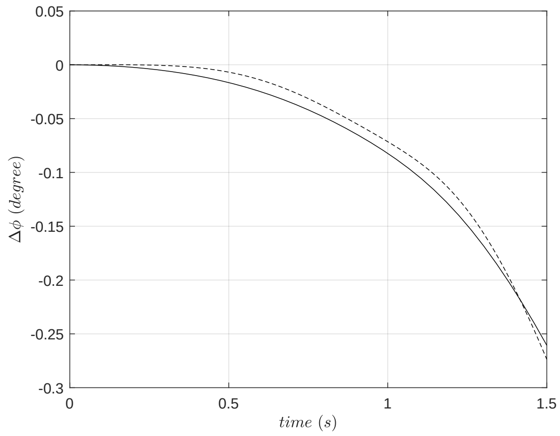

Figure 9 Comparison test. Angular Displacement:
Solid Model(—), Math Model(- -).

Figures 9 and 10 show the angular displacement of the moving platform and the linear displacements of its reference point P obtained either by the porposed model and by the solid model simulation.

5.2 COMPARISON TEST

The dynamic model was validated by comparing the results of a simulation with those obtained by a solid model built by a commercial software (Figure 7). Eventually, Figure 11 shows the comparison between the elongation $\Delta\rho_1$ of the prismatic joint calculated by the two models. From the plots it can be concluded that the models are in good agreement. Despite of the fairly results for the reference point displacement of the moving platform and for the actuators displacements, a discrepancy was found in the moving platform rotation. A possible explanation can be

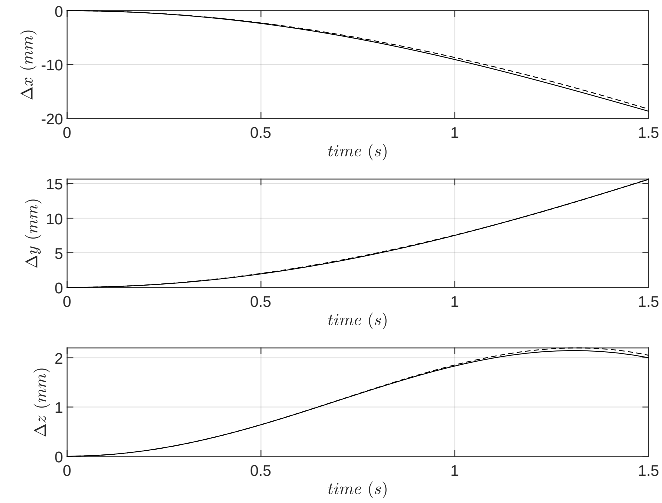

Figure 10 Comparison test. Linear Displacements:
Solid Model(—), Math Model(- -).

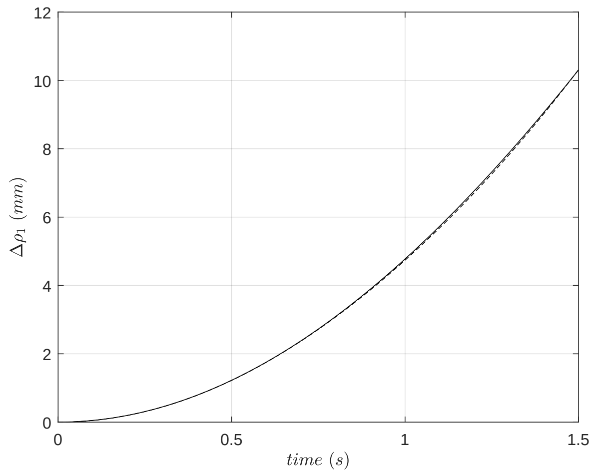

Figure 11 Comparison test. Motor elongation $\Delta\rho_1$:
Solid Model(—), Math Model(- -).

found in the fact that the U joints are immaterial in the mathematical model and their contribution in the rotational inertia may become important for such small value attained by the moving platform rotation.

6 CONCLUSION

A model of the dynamics of a Schönflies motion generator was developed. The model was based on the Lagrange formulation with redundant coordinates comprising either the kinematics of the legs or the moving platform. Both the forward and the inverse dynamic problems were dealt with. The reliability and correctness of the model was tested via the *auto-test*. The forward and inverse computations were, indeed, implemented with reversed input/oputput providing the same results. Furthermore, a validation of the model was carried out by comparing the results from the model with those obtained by a solid model built by a commercial software. The comparison shows a good agreement between the models proving that the model developed is reliable. Moreover, the model is simple and the computational cost is low. The approach used, borrowed from the computational multibody community, can be used for other parallel architectures with small modifications.

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