Analysis of Damage of Ball Bearings of Aeronautical Transmissions by Auto–Power Spectrum Spectrum and Cross–Power Spectrum

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Fourier analysis is used as a framework in which to obtain spectral estimates for non–stationary discrete time signals. Analysis of the auto–power spectrum and the cross–power spectrum makes it possible to distinguish between false alarms and real risk situations and, in the latter case, to determine the propagation of any possible damage in its earliest phases. This analysis then allows corrective actions to be taken, such as parts replacement, in order to limit damage and contamination of component parts in fatigue tests and thus shorten machine stoppage times. [DOI:10.1115/1.1448.320][1]

1 Introduction

Discrete time signals and systems possess both a time domain and a frequency domain, each of which plays an important role in the theory and design of discrete time signal processing systems. In many situations the industrial processes that generate signals are somewhat complex, and modelling the signal as a stochastic process may therefore be analytically useful. Furthermore, many mechanical systems generate acoustic or vibratory signals that can be processed to diagnose potential failure and these are often best modelled as stochastic signals. A stochastic signal is characterised by a set of probability functions and the key to its mathematical representation lies in describing it in terms of mean values. In this paper we perform a statistical analysis of the accelerations acquired by sensors installed on the transmission box called the Intermediate Gearbox (figure 1), which connects the main transmission shaft to the tail rotor of an *EH101* helicopter. The transmission box was mounted on the gear testing stand so that gear fatigue tests could be performed. We consider the spectrum of the ball bearings as the trajectory of a stochastic process. Each spectrum has a different signature, a feature that rules out the possibility of using a deterministic approach which would require equal spectra in the same experimental conditions. As we could not consider equal spectra, we adopted the statistical approach and developed a theory to calculate the waveforms of auto–correlation and cross–correlation functions in a univariate and a multivariate analysis.The mechanism by which high frequency noise is transmitted through the gearbox support struts is rather complex. However, two analytical models proposed by Brennan et al. [1] make it possible to predict the dynamic behaviour of an experimental rig containing an EH101 helicopter gearbox support strut. In order to rank the contributions of the various vibration modes through the strut, the kinetic energy of the receiving structure was calculated from measured data. Signal processing is an extremely important activity as pulse and vibration signals in gears are often associated with impacting faults and can therefore be used as fault indicators. These signals are analysed in the time and frequency domains using a Wigner higher order time–frequency representation [2]. Dellomo [3] considers the feasibility of using a neural network to perform fault detection on vibration measurements given by accelerometer data.

An exact description of the mechanical system is neither simple nor desirable but it is possible to describe a single component of the mechanical system. For example, Ono et al. [4] describe an investigation into bearing vibration caused by the outer race waviness of a ball bearing. This waviness is assumed to exist only on the outer race but not on the inner race. Another theoretical model was presented by Tandon et al. [5] to predict the vibration response of rolling element bearings with distributed defects on the outer race, inner race, or one of the rolling elements. As it is no simple task to describe the analytical model of a mechanical system, signal analysis of the mechanical systems needs to be developed in order to predict damage to ball bearings [6].

[1]Contributed by Technical Committee on Vibration and Sound for pubblication in the JOURNAL OF VIBRATION AND ACOUSTICS. Manuscript receveid November 2000; Revised July 2001; Associate Editor: G.T. Flowers.

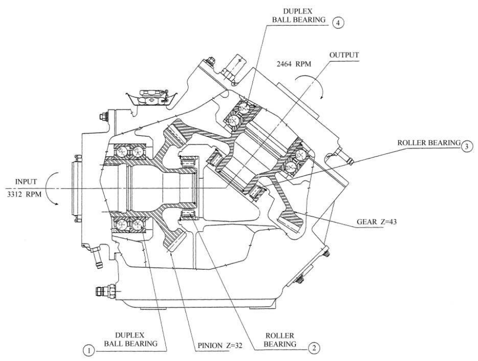

Figure 1: Box of transmission

2 Analysis

There stationary phenomenon in one which the statistical quantities do not change as a function of time. Since we cannot consider all statistical quantities at the same time, a random process is stationary if the mean value of the random process, the autocorrelation function value and the cross correlation function value are independent of the considered moment in time [7]. A nonstationary signal is one for which the signal properties vary with time, for example a sum of sinusoidal components with time–varying amplitudes, frequencies or phases. Typically, when the input to a linear time–invariant system is modelled as a stationary random process, many of the essential input and output characteristics can be adequately represented by mean properties such as the mean values of the auto–power spectrum and the cross–power spectrum. Consequently, these functions of the random process offer tools to examine the evolution of damage phenomena.

According to theorem of Wiener–Khintchine, we compute the single–sided, scaled auto–power spectrum of a time domain signal, assuming a first finite record of a random signal $\boldsymbol{X}[n]$ [8, 9]. The sequence is denoted as

$$V[n] = \begin{cases} \boldsymbol{X}[n] & 0 \le n \le N-1 \\ 0 & \text{otherwise} , \end{cases} \quad (1)$$

where N is the number of points in the signal array $\boldsymbol{X}$. We compute $DFT(\boldsymbol{X})$, the N–point Discrete Fourier Transform of $\boldsymbol{X}[n]$,

$$DFT(\boldsymbol{X}) = \boldsymbol{X}[k] = \sum_{n=0}^{N-1} \boldsymbol{X}[n]\, e^{-j(2\pi/N)kn} , \quad (2)$$

for $k = 0, 1, \ldots, N-1$. If we multiply by $DFT^*(\boldsymbol{X})$, where * denotes the complex conjugate, we obtain $|\boldsymbol{X}[k]|^2$, which corresponds to the circular convolution of finite–length sequence $\boldsymbol{X}[n]$. Then, we calculate the auto–power spectrum as

$$APS = \frac{|\boldsymbol{X}[k]|^2}{N} = \frac{DFT(\boldsymbol{X}) \times DFT^*(\boldsymbol{X})}{N} , \quad (3)$$

In order to calculate the cross-power spectrum of two time domain signals we assume a second finite record of a random signal $\boldsymbol{Y}[n]$. The sequence is denoted as

$$W[n] = \begin{cases} \boldsymbol{Y}[n] & 0 \le n \le N-1 \\ 0 & \text{otherwise} , \end{cases} \quad (4)$$

where N is the number of points in the signal array $\boldsymbol{Y}$. We compute $DFT(\boldsymbol{X})$, the N–point Discrete Fourier Transform of $\boldsymbol{Y}[n]$,

$$DFT(\boldsymbol{Y}) = \boldsymbol{Y}[k] = \sum_{n=0}^{N-1} \boldsymbol{Y}[n]\, e^{-j(2\pi/N)kn} , \quad (5)$$

for $k = 0, 1, \ldots, N-1$. Then we compute the single–sided, scaled cross–power spectrum of two time domain signals. The cross-power spectrum is defined as

$$CPS = \frac{DFT(\boldsymbol{X}) \times DFT(\boldsymbol{Y})}{N} . \quad (6)$$

A typical estimate of the mean value of a stationary random process from a finite length segment of data is the sample mean, defined as

$$I = \frac{1}{N} \sum_{n=0}^{N-1} \boldsymbol{X}[n] . \quad (7)$$

The percent total harmonic distorsion plus noise present in the input auto–power spectrum is computed using the following equation:

$$\%TDH + Noise = \frac{100\sqrt{\sum (APS)^2}}{A(f_1)} . \quad (8)$$

where

- $\sum(APS)$ is the sum of the auto–power spectrum without the power near *DC* and near the fundamental frequency;
- $A(f_1)$ is the amplitude of the fundamental component.

3 Application

With reference to the kinematic diagram in Fig.1, the mechanical system comprises two rotating shafts connected through a toothed couple. The first shaft rotates at a frequency of 56.6 Hz and is mounted on ball bearings denominated *V505* (duplex ball bearing n.1 in Fig.1) and *V502* (roller bearing n.2 in Fig.1), respectively. The second shaft rotates at a frequency of 42.1 Hz and is mounted on ball bearings denominated *V502* (roller bearing n.3 in Fig.1) and *V507* (duplex ball bearing n.4 in fig.1), respectively. The characteristic frequencies of these ball bearings are shown in Table 1. The cogwheel has $z_1 = 32$ teeth and the crown has $z_2 = 343$ teeth. The transmission relationship is 1.3 (see Table 2).

The inner race of the driven wheel's duplex ball bearing presents a defect which consists of spalling over a surface area of $45\mathrm{e}-6\ \mathrm{m}^2$. During fatigue tests we observed a progressive increase in modulation at $1\times$ rev of the crown around the harmonics of the ball bearing's inner race frequency (*IRF* $= 380\mathrm{Hz}$).

The defect on the rotating part consists of spalling over $90\mathrm{e}-6\ \mathrm{m}^2$ of a roller, originating on the side radius and propagating towards the centre of the roller in the bearing mounted on the driving wheel. During fatigue tests we observed a progressive increase in the harmonics of the roller bearing's outer race frequency (*ORF* $= 324$ Hz).

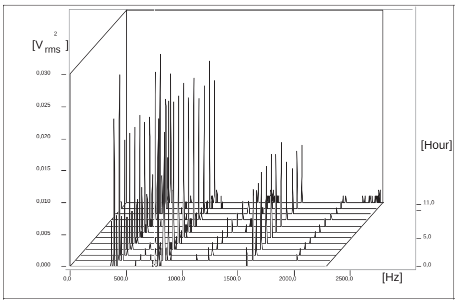

Figure 2: Waterfall of the Discrete Fourier Transform of signal X

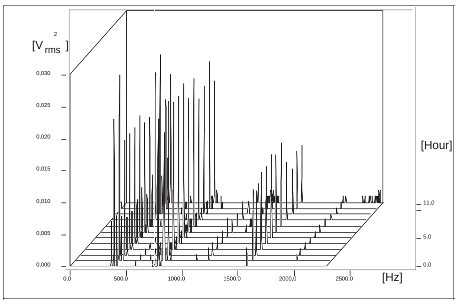

Figure 3: Waterfall of the Auto Power Spectrum of signal X

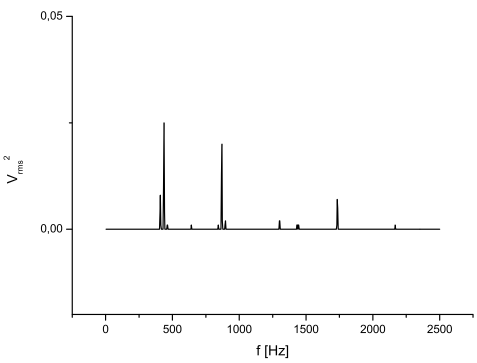

Figure 4: Auto Power Spectrum of ninth hour

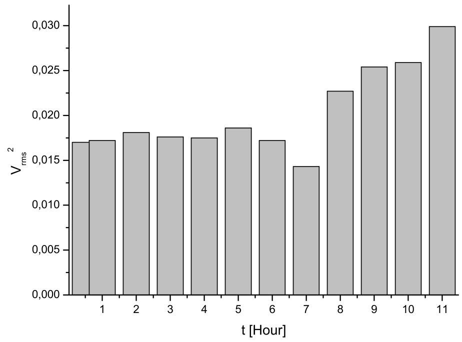

Figure 5: Max Value of APS around 380 Hz

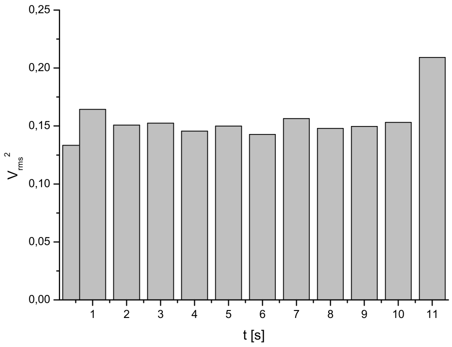

Figure 6: Numerical Integral of the APS of signal

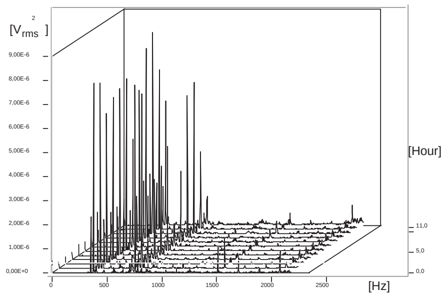

Figure 7: Waterfall of Cross Power Spectrum

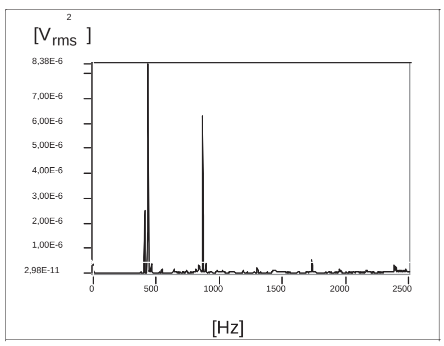

Figure 8: Cross Power Spectrum of ninth hour

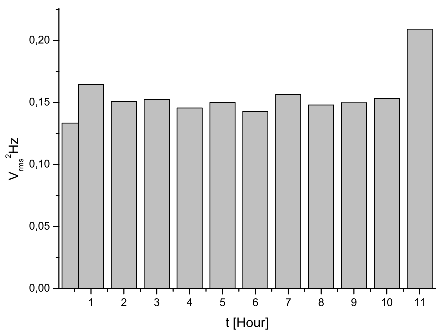

Figure 9: Max Value of CPS around 380 Hz

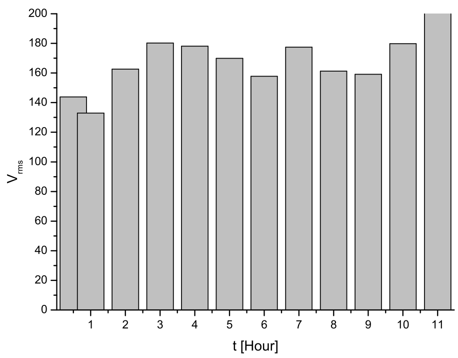

Figure 10: THD+Noise present in the APS

4 Results

With reference to the univariate analysis, we examine the maximum auto–power spectrum values of the whole historical series (Fig.2) from the beginning of the experiment (30 minutes) to the failure of the mechanical system (11 hours). We observe two trends in the maximum auto–power spectrum value: stationary and nonstationary. We observe a non–stationary trend in the auto–power spectrum around the harmonics of the ball bearing's inner race frequency (*IRF* $= 380$ Hz). At the beginning of the experiment we have a maximum auto-power spectrum value of $0.017\ \mathrm{V}^2_{\mathrm{rms}}$. In the phase from the first to the ninth hour of testing we find a stationary trend for the maximum auto-power spectrum value, ranging between 0.017 and $0.019\ \mathrm{V}^2_{\mathrm{rms}}$ (Fig.3). In the proximity of system failure (9 hours) we observe an increase in the maximum auto-power spectrum value to $0.023\ \mathrm{V}^2_{\mathrm{rms}}$ (Fig.4). The set of tests close to system failure shows a non–stationary trend, after the above increase in the maximum auto–power spectrum value (Fig.8).

In other words, we observe an increase in the numerical integration values of auto–power spectrum of the vibration signals. At the beginning, the numerical integration value of the *APS* of X is $0.13\ \mathrm{V}^2_{\mathrm{rms}}\mathrm{Hz}$ (Fig.6).

During the stationary phase, the numerical integration value of the auto–power spectrum of the signal X is constant and about $0.15\ \mathrm{V}^2_{\mathrm{rms}}\mathrm{Hz}$. In proximity of system failure, the numerical integration value of the auto–power spectrum of the signal X is $0.21\ \mathrm{V}^2_{\mathrm{rms}}\mathrm{Hz}$, because the ground noise of the same vibration signal increases. Therefore, the increase in ground noise is related to the incipient failure of the mechanical system. The tendency of the non–stationariness of vibrations, indicated by the function of the auto–correlation factor, could be assumed as a first warning of system failure.

As regards the multivariate analysis, we examine the maximum cross–correlation values for the whole historical series from the beginning of the experiment (30 minutes) to mechanical system failure (11 hours). We calculate the maximum cross–power spectrum values (Fig.7), choosing the reference signal $X = 0.5$ hour and $0.5 \leq Y \leq 11$ hours (Table 3). We observe two trends for the maximum *CPS* value, stationary and non–stationary. We observe a non–stationary trend for the cross–power spectrum around the harmonics of the ball bearing's inner race frequency (*IRF* $= 380$ Hz). At the beginning of the experiment we have a maximum *CPS* value of $7.9\mathrm{e}-6\ \mathrm{V}^2_{\mathrm{rms}}$. In the phase between the first and the ninth hour of tests we observe a stationary trend in the maximum *CPS* value, between $6.0\mathrm{e}-6\ \mathrm{V}^2_{\mathrm{rms}}$ and $7.9\mathrm{e}-6\ \mathrm{V}^2_{\mathrm{rms}}$. In the proximity of system failure (9 hours) we observe an increase in the maximum *CPS* value to $8.4\mathrm{e}-6\ \mathrm{V}^2_{\mathrm{rms}}$ (Fig.10). From the set of tests close to system failure we observe a non–stationary trend, after the above increase in the maximum *CPS* value (Fig.**??**).

The increase in the maximum *CPS* value is caused by an increase in the area subtended to the signal Y. As pointed out in the discussion of *APS*, we observe the increase in ground noise (Fig.9), related to a phenomenon of incipient failure of the mechanical system. Therefore, the tendency of non–stationariness of vibrations, identified by the function of the cross–correlation factor, could be assumed as a second warning of system failure

5 Conclusions

The development of a helicopter transmission system entails an extensive testing procedure if it is to (i) ensure compliance with the design's technical specifications (performance and weight); (ii) meet the standards laid down in the regulations of the agencies issuing civil certification and military qualification; and (iii) assess product maturity (safety and reliability) and guarantee a high *TBO* value (Time Between Overhauls).

During the design development phase, fatigue tests are carried out on the transmission gears in order to check the structural strenght of components subjected to working loads and to identify any differences between one production batch and another. Therefore, from the viewpoint of signal diagnostics verification, the gear fatigue tests are extremely rigorous.

These tests are carried out on a test rig in order to verify the infinite life cycle requisite ($10.000.000$ cycles) for all transmission gears by applying the maximum working load envisaged for the aircraft, duly increased to make allowance for a safety factor, under nominal rotation conditions.

The gear fatigue tests generally give rise to certain problems in other components of the transmissions system (castings, free wheels, bearings) that are not subjected to such tests and this may well jeopardise the possibility of passing the test.

In particular, the bearings are designed for a finite life cycle according to the helicopter's average range of use. Bearing failure depends on the type of failure occuring in the transmission system and will result in the lamination of particles between gear wheel surfaces or the loss of shaft constraint, which will have serious consequences on all adjacent parts.

As mentioned above, fatigue test monitoring is a highly complex operation which is carried out using extremely sophisticated diagnostic and verification instruments and systems. Acceleration, oil temperature and oil pressure in the most critical components, such as bearings subjected to high–speed (turbine intake) or high load (the main rotor shaft bearings) are monitored in real time along with other particular features, including:

- the formation (if any) of metal particles (debris) identified by chip detectors;
- the forces applied to the main rotor shaft;
- the oscillations and variation of the applied torsional load;
- and the vibrations generated by gears and bearings.

These sophisticated systems make it possible to distinguish between false alarms and real risk situations and, in the latter case, to determine the propagation, type and position of any possible damage in its earliest phases so that corrective action can be taken or parts replaced. The aim of this monitoring operation is thus to limit damage and contamination of component parts in fatigue tests and thus shorten machine stoppage times.

The drawbacks identified in the present study regard the damage of two different bearings, a ball bearing (defect in the inner race) and a roller bearing (defect in the rotating element), which occur at two different times in the Intermediate Gearbox (Fig.1). This transmission box links the main transmission shaft to the *EH101* helicopter's tail rotor mounted on the transmission testing stand so that gear fatigue tests could be performed. In our investigation, bearing vibrations were verified using in frequency spectrum analysis of the vibration accelerations acquired by an accelerometer fitted on the box. In particular, the trends of the auto–correlation and cross–correlation factors were recorded. The trends of the auto–correlation and the cross–correlation factors offer two indicators of incipient mechanical failure. The maximum values of the auto–power spectrum and the cross–power spectrum factors increase in the proximity of system failure. A comparison of the auto–power spectrum and cross–power spectrum factors shows that incipient system failure is better indicated by the non–stationary trend of the auto–power spectrum factor. The main features of the method and its limitations of applicability can be summarised as follows:

1. It makes it possible to recognize signal non–stationariness and hence the presence of defects related to the non - linearity of the mechanical system.
2. It identifies the propagation of possible damage in any type of ball bearing.
3. It is not possible to identify the geometric position of the part of the damaged mechanical component.
4. It does not present any limitations of a numerical nature.
5. Levels similar to or higher than 11th hour level are experienced at various times. This severely limits the amount of warning the method is able to provide.

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Ball Bearing	Outer Race Frequency (Hz)	Inner Race Frequency (Hz)	Roll Element Frequency (Hz)	Cage Frequency (Hz)	Type
V505	393.1	512.2	183.1	24.6	Duplex
V502	324.0	468.1	150.5	23.1	Roller
V502	241.2	348.4	112.0	17.2	Roller
V507	292.6	381.2	136.3	18.3	Duplex

Table 1: Characteristic Frequencies

ID	Shaft	N. teeth	RPM	Rot. freq. (Hz)
N1	First	$z_1 = 32$	3394.829	56.6
N2	Second	$z_2 = 43$	2526.384	42.1

Table 2: Cogwheel

Time–Domain Signal X (Hour)	Time–Domain Signal Y (Hour)	Cross–Power Spectrum (Hour)
0.5	0.5	0.5
0.5	1	1
0.5	2	2
0.5	3	3
0.5	4	4
0.5	5	5
0.5	6	6
0.5	7	7
0.5	8	8
0.5	9	9
0.5	10	10
0.5	11	11

Table 3: Cross–Power Spectrum

Nomenclature

$^* =$	Complex coniugate
APS=	Auto Power Spectrum
CPS=	Cross Power spectrum
DFT$(X) =$	Discrete Fourier Transform
$N =$	Number of points
$I =$	Medium value of the random process
$V[n], W[n] =$	Input sequence
$X[n], Y[n] =$	Random signal
$z_1, z_2 =$	Number of teeth
$\%THD+$ noise =	Percent total harmonic distorsion plus noise
$\sum APS =$	Sum of the auto–power spectrum
$A(f_1) =$	Amplitude of the fundamental component