Kinematic Analysis and Centrodes Between Rotating Tool with Reciprocating Motion and Workpiece

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Abstract. A general algorithm for the kinematic analysis and the determination of both fixed and moving centrodes of the relative motion between rotating tool and workpiece, is proposed to analyze the effects of the working parameters on the quality of the machining process. The reciprocating motion of the rotating tool can be generated by both offset slider-crank mechanism with the driving crank and offset slider-rocker mechanism with the driving coupler. The proposed algorithm has been implemented in Matlab and validated by means of significant examples.

Keywords: Rotating tool · Reciprocating motion · Kinematic analysis · Centrodes

1 Introduction

Several machines are provided of rotating tools with a cylindrical shape, in order to perform working processes of different type, as for cutting, milling, grinding, cleaning or washing a given workpiece. Significant examples are the disc saw machine, the milling and grinding machines, the cleaning and washing brush machines. The kinematic analysis of the relative motion between the rotating tool and workpiece is fundamental to set the optimal working parameters of the corresponding machining process.

An analysis of the milling process which considers the relative motion by using the centrodes was proposed in [1] and [2]. In fact, any rigid body motion can be reproduced by the pure-rolling between a pair of centrodes, as shown in [3–8], while the instantaneous geometric invariants were applied in [9, 10]. Another application of the centrodes can be found in [11] where the kinematics of the vehicle motion was analyzed. Instead, the performance of long-dwell planar mechanisms was analyzed in [12]. Axodes and pitch surfaces were also determined for linkages in [13–15] and gears in [16, 17].

In this paper, the case of fixed workpiece and rotating tool that is moved according to a reciprocating motion by an offset slider-crank or rocker mechanism, is considered. Thus, a general algorithm for the kinematic analysis and the determination of both fixed and moving centrodes for the relative motion between rotating tool and workpiece is proposed

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to analyze the effects of the working parameters on the quality of the machining process. In particular, both offset slider-crank mechanism with the driving crank and offset slider-rocker mechanism with the driving coupler, are analyzed in parametric form for different dimensions and input angular velocities of both driving link and rotating tool (2 d.o.f.s). This algorithm has been validated by significant examples.

2 Kinematic Analysis and Centrodes

The kinematic analysis and the determination of both fixed and moving centrodes is developed for the relative motion between a fixed workpiece and a rotating tool, which axis is moved by the piston of an offset slider-crank or rocker mechanism with 2 d.o.f.s.

In particular, the proposed algorithm includes offset slider-crank mechanisms with a driving crank and offset slider-rocker mechanisms with a driving coupler. In both cases, a constant angular velocity ω_2 or ω_3 of the driving link 2 or 3 is assigned, along with ω_5 of the rotating tool 5, which can be different and opposite, as shown in Fig. 1.

The position analysis is developed by the following loop-closure equation

$$\mathbf{r}_1 + \mathbf{r}_4 = \mathbf{r}_2 + \mathbf{r}_3 \tag{1}$$

where $\mathbf{r}_1$ changes in magnitude only, while $\mathbf{r}_4$ is constant since representing the offset.

Consequently, the position vectors $\mathbf{r}_A$, $\mathbf{r}_B$ and $\mathbf{r}_C$ of points A, B and C are given by

$$\mathbf{r}_A = \mathbf{r}_2 \qquad \mathbf{r}_B = \mathbf{r}_2 + \mathbf{r}_3 \qquad \mathbf{r}_C = \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_5 \tag{2}$$

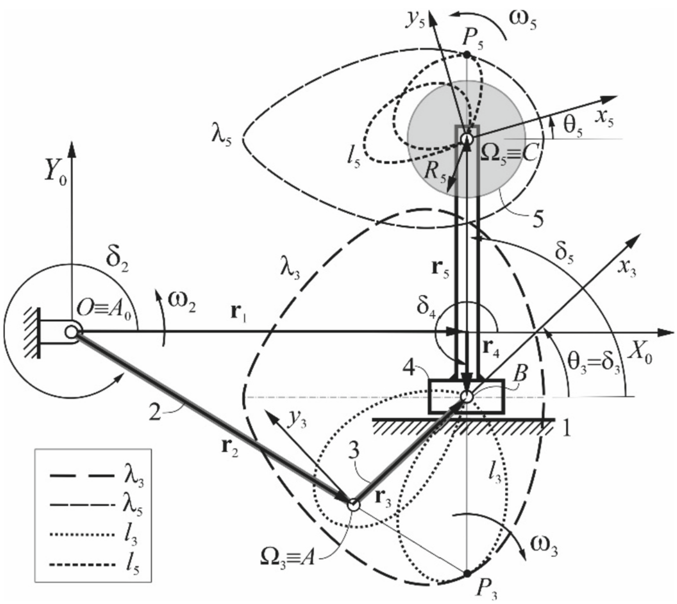

Fig. 1. Kinematic sketch of the slider-crank/rocker mechanism and the centrodes.

Thus, each vector $\mathbf{r}_i$ for $i = 1, \ldots, 5$ can be expressed in matrix form with respect to the fixed frame OX_0Y_0 as

$$\mathbf{r}_i = r_i[\cos\delta_\mathrm{i}\ \sin\delta_\mathrm{i}\ 1]^T \tag{3}$$

where r_i and δ_i are the magnitude and the counterclockwise angle of vector $\mathbf{r}_i$ respectively, while T indicates the transpose.

Developing Eq. (1), angle δ_J can be expressed as

$$\delta_J = \sin^{-1}\left(\frac{r_4 - r_K \sin\delta_K}{r_J}\right) \tag{4}$$

where subscripts K and J are respectively equal to (2) and (3) for the driving crank (2), while they exchange each other in $K = 3$ and $J = 2$ for the driving coupler (3), while the counterclockwise angle θ_5 of the rotating tool (5) can be expressed as

$$\theta_5 = \frac{\omega_5}{\omega_K}\delta_K \tag{5}$$

The first-time derivative of Eq. (4), gives the angular velocity ω_J as follows

$$\omega_J = -\frac{r_K \cos\delta_K}{r_J \cos\delta_J}\omega_K \tag{6}$$

and, in turn, the velocity vectors $\mathbf{v}_A$, $\mathbf{v}_B$ and $\mathbf{v}_C$ of points A, B and C are given by

$$\mathbf{v}_A = [-r_2\omega_2 \sin\delta_2,\ amp; r_2\omega_2 \cos\delta_2]^T, \quad \mathbf{v}_B = [-r_2\omega_2 \sin\delta_2 - r_3\omega_3 \sin\delta_3,\ amp; 0]^T, \\ \mathbf{v}_C = \mathbf{v}_B \tag{7}$$

In general, the relative planar motion between two rigid bodies can be reproduced by the pure rolling of two curves that take the role of centrodes, which are traced by the instantaneous center of rotation on their corresponding planes. If one of the two rigid bodies is fixed, a pair of fixed and moving centrodes can be obtained.

In particular, referring to Fig. 1, the fixed centrode λ_3 is the path traced by the instantaneous center of rotation P_3 of the coupler link 3 with respect the fixed frame OX_0Y_0, while the moving centrode l_3 is the path traced by P_3 with respect the moving frame $\Omega_3x_3y_3$ that is attached to AB.

According to the Aronhold-Kennedy theorem, the fixed centrode λ_3 of the coupler link 3 can be expressed by

$$\mathbf{r}_{P3} = [r_2 \cos\delta_2 + r_3 \cos\delta_3\ \ r_2 \sin\delta_2 + \mathrm{r}_3 \cos\delta_3 \mathrm{tg}\delta_2\ \ 1]^T \tag{8}$$

and, likewise, the moving centrode l_3 takes the form

$$\mathbf{r}^*_{P3} = r_3\frac{\cos\delta_3}{\cos\delta_2}[\cos(\delta_2 - \delta_3)\ \sin(\delta_2 - \delta_3)\ 1]^T \tag{9}$$

Similarly, the instantaneous center of rotation P_5 for the absolute motion of the rotating tool 5 with respect to the fixed frame OX_0Y_0, must be aligned along the normal line to the straight trajectory of point B of the piston (4), with both instantaneous centers of rotation P_4 and P_{45}, which are improper and proper points, respectively.

Nevertheless, this condition of the Aronhold-Kennedy theorem is not sufficient to define the unique position of P_5, because the whole mechanism that includes the rotating tool, has 2 d.o.f.s. In fact, the angular velocity ω_5 is also given as second kinematic input data, along with the angular velocity ω_2 or ω_3 for the cases of driving crank or driving coupler, respectively.

Consequently, the fixed centrode λ_5 of rotating tool (5) is given in vector form as

$$\mathbf{r}_{P5} = \left[r_2 \cos\delta_2 + r_3 \cos\delta_3 \;\; r_2 \sin\delta_2 + r_3 \cos\delta_3 + r_5 + \tfrac{v_B}{\omega_5} \;\; 1 \right]^T \tag{10}$$

and, in turn, the moving centrode l_5 is given by

$$\mathbf{r}^*_{P5} = \frac{v_B}{\omega_5}[\, \sin\theta_5 \;\; \cos\theta_5 \;\; 1\,]^T \tag{11}$$

In order to express both moving centrodes l_3 and l_5 of the coupler link (3) and the rotating tool (5) with respect to OX_0Y_0, the position vector $\mathbf{r}^*_{Pi}$ of a generic point P with respect to the moving frame $\Omega_i\, x_i\, y_i$ can be expressed as

$$\mathbf{r}^*_{Pi} = r^*_{Pi}\left[\cos\theta_\mathrm{i} \;\; \sin\theta_\mathrm{i} \;\; 1 \right]^T \tag{12}$$

and thus, the position vector $\mathbf{r}_{Pi}$ of P in OX_0Y_0 can be given

$$\mathbf{r}_{Pi} = \mathbf{T}_{0\mathrm{i}}\, \mathbf{r}^*_{Pi} \tag{13}$$

being $\mathbf{T}_{0i}$ the following transformation matrix by the moving frame $\Omega_i\, x_i\, y_i$ to OX_0Y_0

$$\mathbf{T}_{0\mathrm{i}} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & x_{\Omega i} \\ \sin\theta_i & \cos\theta_i & y_{\Omega i} \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

where θ_i is the counterclockwise rotation angle of $\Omega_i\, x_i\, y_i$ with respect to OX_0Y_0, while $x_{\Omega i}$ and $y_{\Omega i}$ are the Cartesian coordinates of the origin Ω_i in the same fixed frame.

3 Numerical Examples and Validation

The proposed algorithm has been implemented in Matlab and validated by means of significant examples, where both cases of rotating tool that is moved in reciprocating motion by slider-crank mechanism and slider-rocker mechanism of several dimensions, along with different kinematic inputs, have been considered.

In particular, Fig. 2 is for the offset slider-crank mechanism with $\delta_2 = 45$, $\omega_2 = 1$ r/s, $\omega_5 = 2$ r/s, $r_2 = 200$ mm, $r_3 = 600$ mm, $r_4 = -50$ mm, $R_5 = 150$ mm and $r_4 = 300$ mm; Fig. 3a is for the offset slider-rocker mechanism with $\delta_3 = 35$, $\omega_3 = 1$ r/s, $\omega_5 = 0.7$ r/s, $r_2 = 600$ mm, $r_3 = 200$ mm, $r_4 = 50$ mm, $R_5 = 150$ mm, $r_5 = 300$ mm; Fig. 3b is the same as Fig. 3a, but with $\omega_5 = 2$ r/s. One can note that referring to Figs. 2 and 3, point P_5 moves toward the center C of the rotating tool by increasing the value of ω_5.

Moreover, the shapes and sizes of λ_5 and l_5 change, while both centrodes λ_3 and l_3 are open or closed curves for slider-crank or slider-rocker mechanisms, respectively.

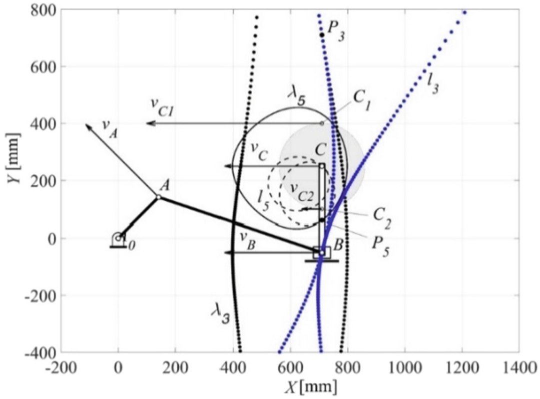

Fig. 2. Offset slider-crank mechanism for $\delta_2 = 45°$, $\omega_2 = 1$ r/s and $\omega_5 = 2$ r/s.

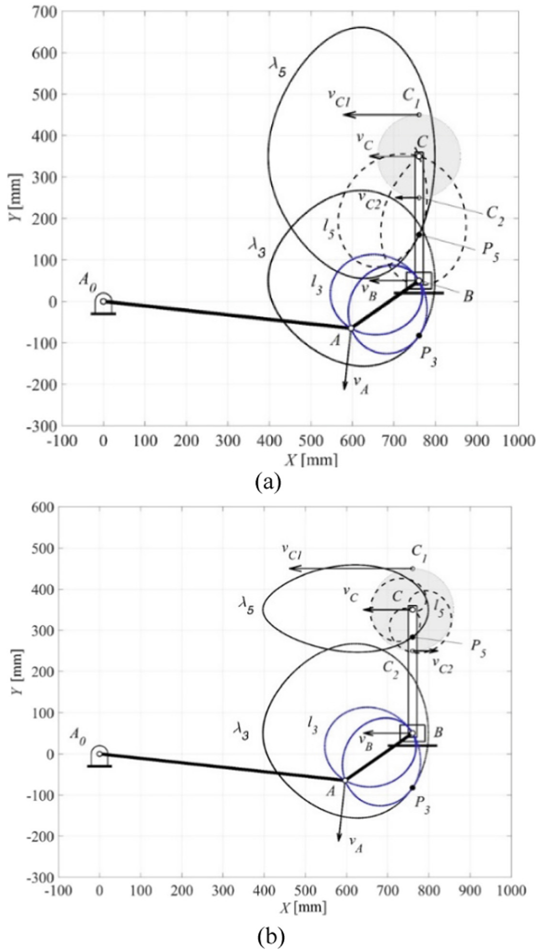

Fig. 3. Offset slider-rocker mechanism for $\delta_3 = 35°$, $\omega_3 = 1$ r/s: a) $\omega_5 = 0.7$ r/s; b) $\omega_5 = 2$ r/s.

4 Conclusions

A general algorithm for the kinematic analysis and the determination of both fixed and moving centrodes for the relative motion between a generic rotating tool and a workpiece has been formulated and validated through significant examples. The centrodes of both coupler link and rotating tool can be obtained for different dimensions of the operating mechanism and any input of both constant angular velocities (2 d.o.f.s).

The proposed algorithm is considered to be useful for analysis and design purposes of all machines that makes use of rotating tools with reciprocating motion, since the centrodes allows the visualization of the velocity vector field and the analysis of their relative motion to set the cutting or working parameters of the machining process.

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