



25th International Conference on Fracture and Structural Integrity

A constitutive model to predict the pseudo-elastic stress-strain behaviour of SMA

Costanzo Bellini^{1*} and Stefano Natali²

¹Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, via G. Di Biasio 43, Cassino 03043, Italy

²Department of Chemical Engineering, Materials and Environment, Sapienza University of Rome, via Eudossiana 18, Roma 00184, Italy

Abstract

Shape memory alloys (SMAs) are a wide class of materials characterized by the property to recover the initial shape also after high values of deformations. This is due to the ability of SMAs to change, in a reversible manner, their microstructure from an initial structure, often named austenite, to a final structure, named martensite. The transformations of microstructure can take place with or without one or more intermediate phases, but always without re-crystallization, implying a microstructure changing inside the crystals, without any new boundary creation. The stress-strain behaviour depends on the crystal structures. In this work, a simple model to predict the stress-strain behaviour of a PE SMA has been proposed. The results have been compared to an experimental tensile test carried out on a NiTi SMA alloy.

© 2019 The Authors. Published by Elsevier B.V.

Peer-review under responsibility of the Gruppo Italiano Frattura (IGF) ExCo.

Keywords: Ductile cast iron; hot dip galvanizing; intermetallic phases.

1. Introduction

Shape memory alloys (SMAs) are a wide class of materials characterized by the ability to remember the initial geometry also after high values of deformations as indicated by Otsuka et al. (2005) and by Eggeler et al. (2004). This is due to the ability of SMAs to change the initial microstructure to a final microstructure under load effect. Lagoudas et al. (2009), studying the influence of temperature on fatigue behaviour of SMA, highlighted that the microstructure

* Corresponding author. Tel.: +39-0776-2993698; fax: +39-0776-2993886.

E-mail address: costanzo.bellini@unicas.it

transformations take place at low temperature without any new boundary origin. It means that the transformations occur inside the grains and no new grains are born during the transformation. Furthermore, these transformations are reversible, allowing to recover the initial shape just recovering the initial structure. For this, the SMAs are characterized by a typical reversible microstructure transformation without any recrystallizations; in one word, the SMAs are characterized by a transition of their microstructure.

Nomenclature

SMA	Shape memory alloy
PE	Pseudo-elastic
E_A	Young's modulus of austenite
E_M	Young's modulus of martensite
K_A	Coefficient of transforming austenite
K_M	Coefficient of transforming martensite

In terms of mechanical behaviour, the SMAs are characterized by different stages (Fig. 1). The first one is the stage where the austenite is stable, and it is characterized by a linear elastic behaviour with Young's modulus of the austenite. The first stage is followed by the second stage, where the microstructure changes from austenite to martensite (with or without intermediate microstructures). The second stage is often schematized as a plateau with a linear microstructure transformation, but many experimental evidences showed that at the beginning of the second stage the stress decreases, then increase with an increasing slope up to the slope of the third stage, where all the austenite transforms in the martensite. Over the third stage, the SMA shows the traditional plasticity stages up to the failure.

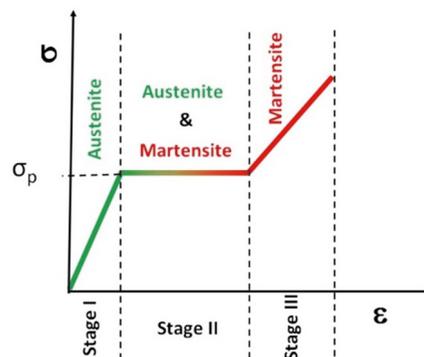


Fig. 1. Schematization of three stress-strain stages.

Some kinds of SMAs are the copper-based SMAs (Natali et al. 2014 and Brotzu et al. 2018) or many iron-based alloys. Due to its high performance, the most important SMA class is the NiTi alloys as indicate in Iacoviello et al. (2014). In scientific literature there are many mechanical approaches to describe the stress-strain response of NiTi SMAs as in Auricchio et al. (1997), and some approaches on fatigue damage prediction, as proposed by Furguele et al. (2012), Kollerov et al. (2013) and Maletta et al. (2017), but there are just a few models which take into account the role of the microstructure evolution on the mechanical behaviour.

For instance, Miyazaki et al. (1989), Kang et al. (2012) and Sgambitterra et al. (2014), proposed an interesting energy approach to describe the cyclic behaviour of a NiTi SMAs, taking into account the ratcheting effect.

In this work, a model proposed by Di Cocco et al. (2018), able to predict the microstructure evolution, has been used in order to calculate the microstructure evolution during a single tensile test in the first three stages. Then a simple stress-strain model, able to take into account the real contribution of austenite and martensite in the mechanical

strength, has been proposed in order to predict the stress response of a NiTi SMA, including the second stage, that is the most critical to be simulated.

2. Material and methods

In this work, a near-equiatomic NiTi SMA, characterized by a pseudo-elastic behaviour, has been used to realize mini flat dog-bone tensile specimens in order to evaluate the stress-strain behaviour by means of a patented tool, suitable for the use in a diffraction test devices (Fig. 2). This tool allows performing X-Ray diffraction tests on the calibrated length of the specimens under load conditions, in order to evaluate the real microstructure (quantification of austenite and martensite).

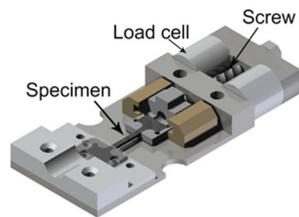


Fig. 2. Patented tool used to perform tensile tests and x-Ray diffractions.

Results of tensile tests have been analysed in terms of engineering stress and engineering strain. Furthermore, a commercial FEM code has been used to implement the proposed mechanical model in order to compare the predicted stress-strain curve to the test results. The tensile tests are simulated by the FEM code, including the evaluation of austenite/martensite ratio calculated by the structural model proposed by Di Cocco et al. (2018).

3. Experimental results

The first three stages of the engineering stress-strain curve, taken by means of step by step procedure in order to perform the X-Ray diffractions, are shown in Fig. 3. The first stage is characterized by the presence of a fully austenitic phase, and the mechanical behaviour is similar to the elastic stage of traditional metallic alloys. The second stage is characterized by a sharp decrease of the slope, followed by an increase of slope up to the martensite Young's modulus.

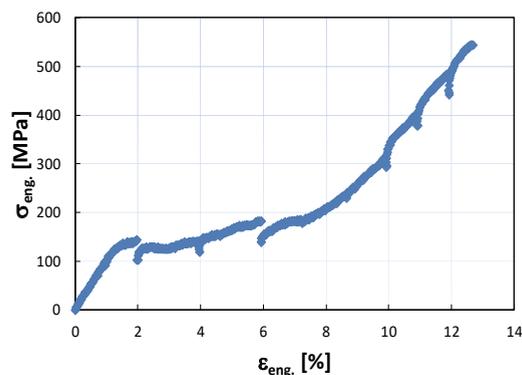


Fig. 3. Engineering Stress-Strain curve.

In terms of microstructure, the first stage is characterized by the spectrum shown in Fig. 4, where it is possible to observe the evaluated positions of the lattice atoms.

Microstructure at $\epsilon_{eng.}=0\%$: Austenite

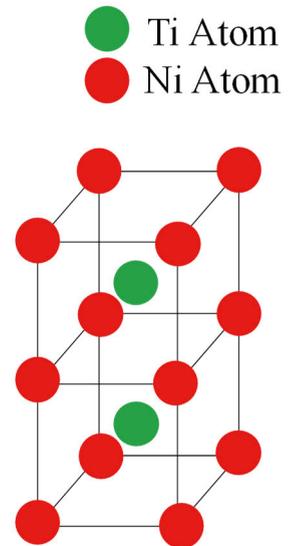
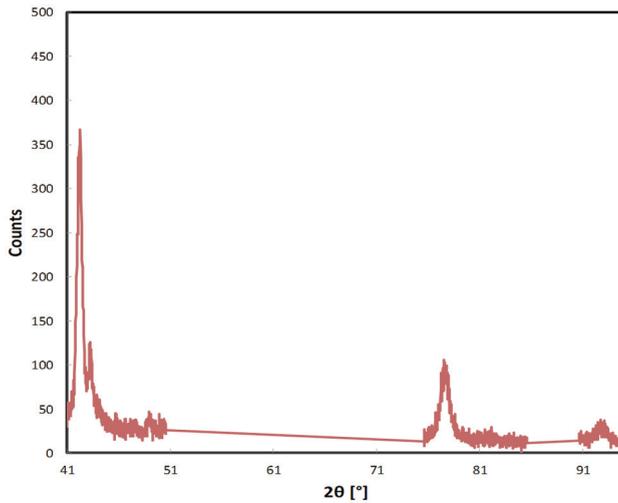


Fig. 4. Diffraction spectrum of austenite and schema of atoms position.

However, in Fig 5 the spectrum of martensite shows the different position of peaks angles due to different microstructure. The position of atoms in the lattice is shown in the martensite schema in Fig. 5.

Microstructure at $\epsilon_{eng.}=10\%$: Martensite

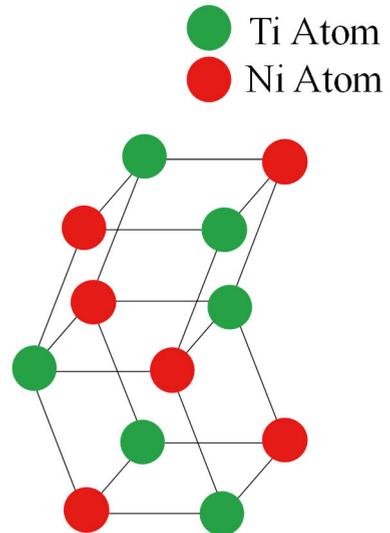
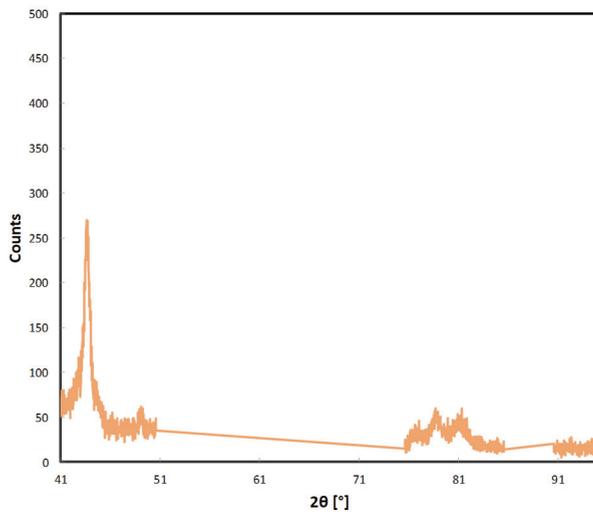


Fig. 5. Diffraction spectrum of martensite and schema of atoms position.

The evaluation of cell parameters using the Bragg law is performed on austenite lattice at the end of the first stage (maximum elastic deformation of austenite) and on the martensite at the beginning of the third stage.

The results are schematized in Fig. 6, where the presence of a deformation excess during the austenite-martensite transformation could be the reason of the sharp decrease of the stress-strain curve slope at the begin of the second stage.

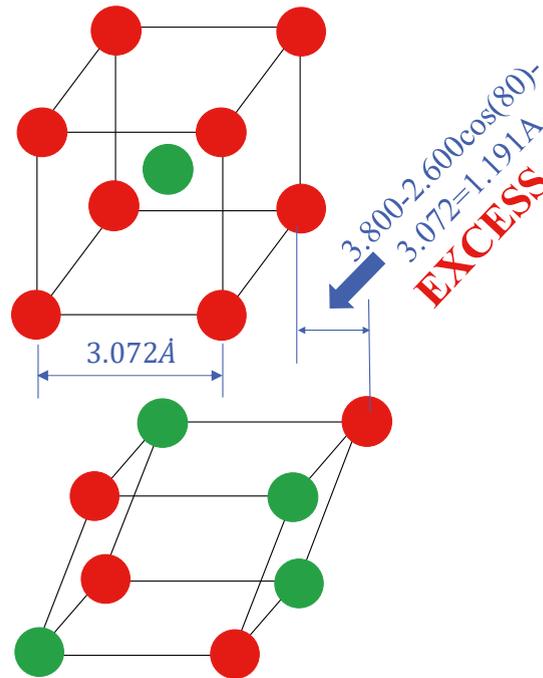


Fig. 6. Evaluation of cell parameters of austenite and martensite under load.

Considering the model proposed by Di Cocco et al. (2014) to predict the microstructure evolution, it is possible to calculate the volume fraction of austenite and martensite respectively by the equations (1) and (2).

$$A = \frac{D e^{-C\varepsilon}}{1 + D e^{-C\varepsilon}} \quad (1)$$

$$M = 1 - A = \frac{1}{1 + D e^{-C\varepsilon}} \quad (2)$$

where the parameters $D=700$ and $C=1$.

In order to evaluate the first three stages of stress-strain curve, a simple model is proposed where the contributions of austenite and martensite are weighed by two different functions as shown in equation (3)

$$E = \frac{A}{1+K_A \frac{\partial A}{\partial \varepsilon}} E_A + \frac{M}{1+K_M \frac{\partial M}{\partial \varepsilon}} E_M \tag{3}$$

where A and M are obtained respectively by equations (1) and (2), and K_A and K_M are two measuring coefficients which consider the effect of transforming austenite (derivative of austenite function) and of transforming martensite (derivative of martensite function). E_A and E_M are Young’s modulus of the austenite and the martensite.

In this case, the parameters are shown in table 1.

Table 1: Austenite and Martensite Parameters.

Austenite		Martensite	
E_A	K_A	E_M	K_M
60 [GPa]	-1250 [MPa ⁻¹]	28 [GPa]	60 [MPa ⁻¹]

The evaluation of the first three stress-strain curve stages is shown in Fig. 7, where the measures of austenite and martensite volume fraction are plotted for the same strain range.

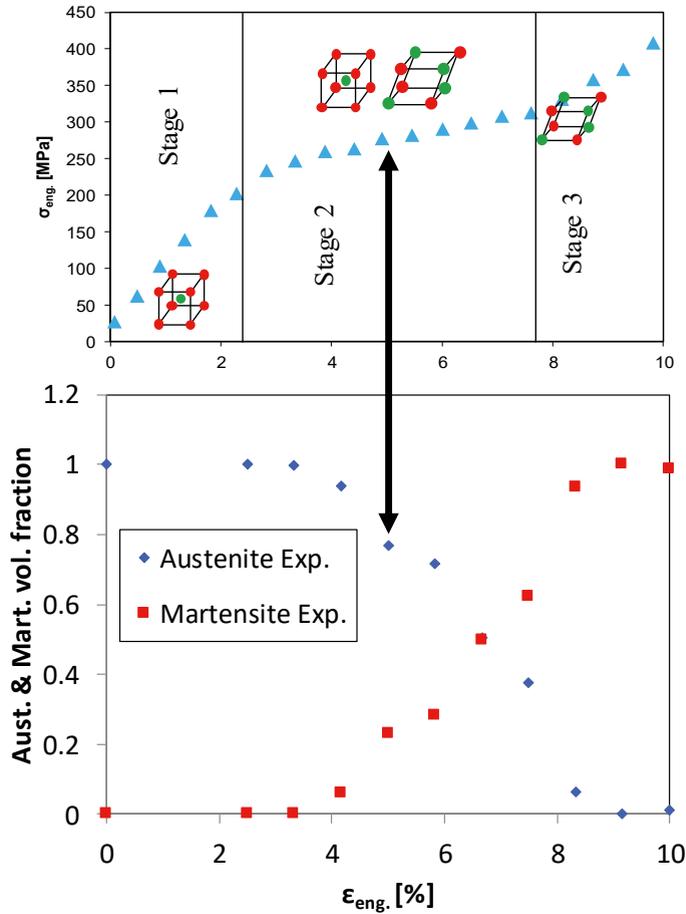


Fig. 7. Evaluation first three stages of the engineering stress-strain curve and the measures of austenite and martensite volume fractions.

It is possible to observe how at the middle of the second stage the volume fractions of austenite and of martensite are not 50%, as often assumed, but the volume fraction of austenite is close to 78%. The 50% of austenite and martensite is obtained near to the end of the second stage.

Finally, in Fig 8, the results of FEM simulation of tensile stress are compared to the measured loads. It is possible to observe the good agreement between calculated loads and the experimental measures. In particular, in the first and in the third stages experimental and numerical curves are coincident, while in the second stage the difference is a bit larger; however, the discrepancy is undoubtedly acceptable, since it is less than 5%.

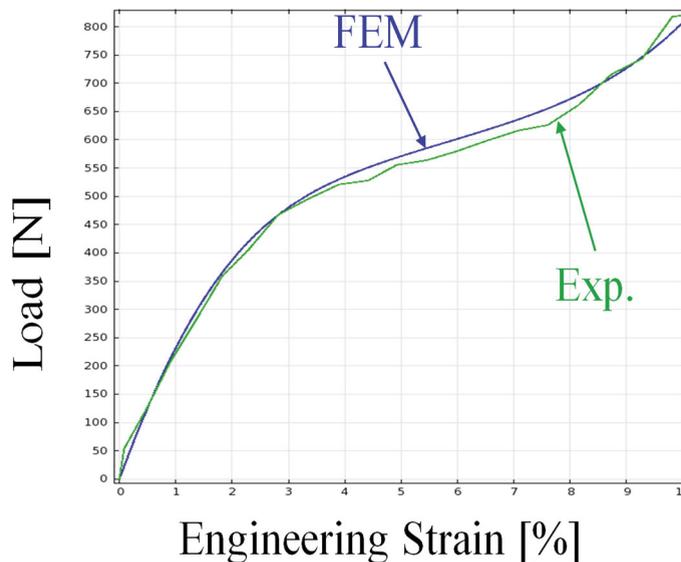


Fig. 8. Comparison between simulated tensile test and experimental results.

4. Conclusions

In this work, a near-equiatomic NiTi SMA characterized by a pseudo-elastic effect is investigated in order to evaluate the tensile behaviour taking into account the microstructure evolutions. A simple constitutive model has been proposed in order to predict the first three stages starting from the austenite/martensite volume fractions.

The main results can be summarized as follows:

- 1) The second stage of stress-strain curve it is not a perfect plateau, but presents a sharp decreasing of the curve slope, probably due to an excess of the cells dimension of martensite compared to the austenite cells.
- 2) The role of austenite/martensite phases transforming is important on the stress-strain behaviour during the tensile test;
- 3) A simple stress-strain model has been proposed, able to predict the first three stages of the stress-strain curve, taking in to account the volume fraction of austenite and martensite.
- 4) The austenite-martensite transformation in the second stage is not linear, reaching 50% over the middle strain range of the second stage.

References

- Auricchio, F., Sacco, E., 1997. A One-Dimensional Model for Superelastic Shape Memory Alloys with Different Elastic Properties Between Austenite and Martensite. *International Journal of Non-Linear Mechanics* 32, 1101.

- Brotzu, A., Iacoviello, F., Di Cocco, V., Natali, S., 2018. Grain Size and Loading Conditions Influence on Fatigue Crack Propagation in a Cu-Zn-Al Shape Memory Alloy. *International Journal of Fatigue* 115, 27. DOI: 10.1016/j.ijfatigue.2018.06.039.
- Di Cocco, V., Natali, S., 2018. A Simple Model to Calculate the Microstructure Evolution in a NiTi SMA. *Frattura ed Integrità Strutturale* 44, 173. DOI: 10.3221/IGF-ESIS.44.14
- Eggeler, G., Hornbogen, E., Yawny, A., Heckmann, A., Wagner, M., 2004. Structural and Functional Fatigue of NiTi Shape Memory Alloys. *Materials Science and Engineering A* 378, 24. DOI: 10.1016/j.msea.2003.10.327
- Furgiuele, F., Maletta, C., Sgambitterra, E., Casati, R., Tuissi, A., 2012. Fatigue of Pseudoelastic NiTi Within the Stress-Induced Transformation Regime: a Modified Coffin–Manson Approach. *Smart Materials and Structures* 21, 112001. DOI: 10.1088/0964-1726/21/11/112001
- Iacoviello, F., Di Cocco, V., Maletta, C., Natali, S., 2014. Cyclic microstructural transitions and fracture micromechanisms in a near equiatomic NiTi alloy, *International Journal of Fatigue* 58, 136. DOI: 10.1016/j.ijfatigue.2013.03.009.
- Kang, G., Kan, Q., Yu, C., Song, D., Liu, Y., 2012. Whole-Life Transformation Ratchetting and Fatigue of Super-Elastic NiTi Alloy under Uniaxial Stress-Controlled Cyclic Loading. *Materials Science and Engineering A* 535, 228. DOI: 10.1016/j.msea.2011.12.071
- Kollerov, M., Lukina, E., Gusev, D., Mason, P., Wagstaff, P., 2013. Impact of Material Structure on the Fatigue Behaviour of NiTi Leading to a Modified Coffin-Manson Equation. *Materials Science and Engineering A* 585, 356. DOI: 10.1016/j.msea.2013.07.072
- Lagoudas, D.C., Miller, D.A., Rong, L., Kumar, P.K., 2009. Thermomechanical Fatigue of Shape Memory Alloys. *Smart Materials and Structures* 18, 085021. DOI: 10.1088/0964-1726/18/8/085021
- Maletta, C., Niccoli, F., Sgambitterra, E., Furgiuele, F., 2017. Analysis of Fatigue Damage in Shape Memory Alloys by Nanoindentation, *Materials Science & Engineering A* 684, 335. DOI: 10.1016/j.msea.2016.12.003
- Miyazaki, S., Imai, T., Igo, Y., Otsuka, K., 1986. Effect of Cycling Deformation on the Pseudoelasticity Characteristic of Ni–Ti Alloys. *Metallurgical Transactions A* 17, 115. DOI: 10.1007/BF02644447
- Natali, S., Di Cocco, V., Iacoviello, F., Volpe, V., 2014. Fatigue Crack Behavior on a Cu-Zn-Al SMA, *Frattura ed Integrità Strutturale* 8, 454. DOI: 10.3221/IGF-ESIS.30.55
- Otsuka, K., Ren, X., 2005. Physical Metallurgy of TiNi-Based Shape Memory Alloys. *Progress in Material Science* 50, 511. DOI: 10.1016/j.pmatsci.2004.10.001
- Sgambitterra, E., Maletta, C., Furgiuele, F., Casati, R., Tuissi, A., 2014. Fatigue Properties of a Pseudoelastic NiTi Alloy: Strain Ratchetting and Hysteresis under Cyclic Tensile Loading. *International Journal of Fatigue* 66, 78. DOI: 10.1016/j.ijfatigue.2014.03.011
- Vantadori, S., Carpinteri, A., Di Cocco, V., Iacoviello, F., Natali, S., 2018. Fatigue Analysis of a Near-Equiatomic Pseudo-Elastic NiTi SMA. *Theoretical and Applied Fracture Mechanics* 94, 110. DOI: 10.1016/j.tafmec.2018.01.012