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A Bayesian Compressive Sensing Approach to Robust Near-Field Antenna Characterization
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Abstract—A novel probabilistic sparsity-promoting method for robust near-field (NF) antenna characterization is proposed. It leverages on the measurements-by-design (MebD) paradigm, and it exploits some a priori information on the antenna under test (AUT) to generate an overcomplete representation basis. Accordingly, the problem at hand is reformulated in a compressive sensing (CS) framework as the retrieval of a maximally sparse distribution (with respect to the overcomplete basis) from a reduced set of measured data, and then, it is solved by means of a Bayesian strategy. Representative numerical results are presented to, also comparatively, assess the effectiveness of the proposed approach in reducing the “burden/cost” of the acquisition process and mitigate (possible) truncation errors when dealing with space-constrained probing systems.

Index Terms—Antenna measurements, antenna qualification, compressive sensing (CS), near-field (NF) pattern estimation, near-field to far-field (NF-FF) transformation, sparsity retrieval, truncation error.

I. INTRODUCTION

We are nowadays witnessing an extraordinary technological advancement in phased array technology as a key asset to the forthcoming 5G and 6G communications standards [1], [2]. High-performance multi-input/multi-output (MIMO), cognitive, and multibeam architectures will undergo mass production to allow an ubiquitous implementation of the Internet-of-Things (IoT)-based next-generation wireless environments [3]. For a fast and reliable antenna certification at the end of large-scale manufacturing processes [4], [5], over-the-air measurements clearly constitute the most time/cost-effective option. In such a framework, far-field (FF) techniques are one viable and consolidated approach. However, they intrinsically suffer from limitations imposed by outdoor sites, such as the vulnerability to weather conditions and to reflections from ( uncontrollable) environmental obstacles/scatterers. Otherwise, near-field (NF) probing methods are alternative solutions that guarantee a higher accuracy and repeatability, thanks to the exploitation of fully controlled indoor environments and the availability of efficient NF-FF transformation strategies [6]–[18], even though they are prone to the so-called “truncation error” caused by the limited extension of the scanning surface (e.g., planar [14], [17], cylindrical [11], spherical [8], and conical [12]) in real anechoic chambers. Moreover, the IEEE recommended practice for NF measurements [19] states that a reliable assessment of the radiation features of an antenna under test (AUT) needs a dense probing step \( \Delta \rho \) (i.e., \( \Delta \rho \leq (\lambda/2) \), with \( \lambda \) being the free-space wavelength). This results in time-consuming acquisition procedures due to the huge number of scanning positions [7]. An effective recipe to avoid/mitigate such issues is to exploit the available a priori information on the AUT. As a matter of fact, several methodologies are based on the representation of the radiation behavior of the AUT in terms of a set of known basis functions defined with accurate full-wave (FW) simulations of the CAD models of the antenna, which are typically available from previous stages of the design process [7], [8]. Accordingly, the NF recovery problem at hand is then reformulated as the retrieval, from a reduced set of measured data, of the expansion coefficients by means of suitable matching strategies [7] or machine learning tools [8]. Within this line of reasoning, the measurements-by-design (MeBd) paradigm has been proposed as an effective tool to predict the AUT features by exploiting, unlike abovementioned state-of-the-art strategies, the generation of an overcomplete basis rather than a minimum-redundancy one [20]–[22]. Thanks to this, it is possible to recast the problem at hand as a sparsity-retrieval one suitable for a fruitful exploitation of the compressive sensing (CS) [20], [23]. As for this latter, it is worth pointing out that a reliable application of standard (deterministic) CS solvers requires a preliminary check of the restricted isometry property (RIP) of the observation operator, which rapidly becomes computationally unaffordable even for small-medium-scale problems [23]. To overcome this issue, the MeBd is mathematically reformulated in this communication within a probabilistic sparsity-promoting framework and then solved by means of a customized Bayesian CS (BSC) strategy that avoids cumbersome assessments of the RIP compliancy [24]. To the best of the authors’ knowledge, the main novelties of this research work lie in: 1) the formulation of the NF prediction problem within a highly flexible Bayesian framework independent on neither the knowledge of the nominal/gold antenna nor on a particular topology of the probing setup and 2) a suitable customization of the BCS to yield a robust and reliable solution of the NF field estimation in a wide range of applicative scenarios.

The outline of this communication is given as follows. The mathematical formulation of the NF antenna characterization problem...
II. MATHEMATICAL FORMULATION

To faithfully retrieve the FF pattern features of an AUT by means of NF-FF transformation rules [18], the radiated tangential electric field distribution must be accurately estimated over a sufficiently large surface Ψ (see Fig. 1) to limit as much as possible the so-called “truncation error” [16]. Generally speaking, Ψ is sampled according to Nyquist’s rule by choosing Δρ = (λ/2) (Δρ being the sampling rate along a generic direction of the surface Ψ) [19] for yielding the set of T locations \( T = \{ r_t \in \Psi; t = 1, \ldots, T \} \). Let us express the NF distribution in \( T \), \( \vec{E} = \{ \vec{E}(r_t); t = 1, \ldots, T \} \), as the linear combination of \( B \) properly built basis vectors, \( \vec{A} = [\vec{A}_1; b = 1, \ldots, B] \), \( \vec{A}_b = [\vec{A}_b(r_t); t = 1, \ldots, T] \) being the \( b \)th \( (b = 1, \ldots, B) \) one through a set of unknown coefficients \( \vec{w} = \{w_b \in \mathbb{C}; b = 1, \ldots, B \} \) \( \vec{E} = \vec{A}\vec{w}. \) According to the MeB paradigm [20], the basis \( \vec{A} \) is defined by exploiting the a priori information on the AUT through the following procedure:

1) Uncertainty Identification: Identify the set of \( C \) uncertainty factors that can cause a deviation of the AUT radiation features from the ideal/gold ones (e.g., defects of the beamforming network and manufacturing tolerances). For each \( c \)th \( (c = 1, \ldots, C) \) uncertainty descriptor, \( \chi_c \) defines a suitable (physically admissible) variation range \( \{\chi_c^{\min}, \chi_c^{\max}\} \). Finally, let \( c = 1 \) and \( b = 0 \), and go to Step 2.

2) Overcomplete Basis Generation: Loop \( (c = 1, \ldots, C) \).

a) Uniformly sample the \( c \)th descriptor to form the set of \( K_c \) configurations \( \chi_c = \{\chi_c^{(k)}; k = 1, \ldots, K_c\} \), with the \( k \)th one being \( \chi_c^{(k)} = \chi_c^{\min} + (k - 1) \frac{(\chi_c^{\max} - \chi_c^{\min})}{(K_c - 1)}. \)

b) Run \( K_c \) FW simulations of the AUT to fill the set of NF distributions \( \vec{E}_c = [\vec{E}_c^{(k)}; k = 1, \ldots, K_c], \vec{E}_c^{(k)} = \{\vec{E}(r_t); t = 1, \ldots, T\} \) being the sampled NF distribution in \( \Psi \) for an AUT whose \( c \)th uncertainty descriptor has a value equal to the \( k \)th sample of its variation range \( (i.e., \chi_c = \chi_c^{(k)}) \).

c) Apply the truncated singular value decomposition (TSVD) to \( \vec{E}_c \)
\[
\vec{E}_c = \vec{U} \Sigma (\vec{V})^*.
\]
As for the NF setup, the nominal/gold excitations being set to excited clusters/planks corresponding to the rows of the array, the feeding architecture consists of S\[10\] sampled square lattice on a hand, this section is aimed at providing the interested readers/users with some useful guidelines for its optimal application.

As for the results, besides a pictorial representation of the NF field reconstructions, the NF integral error

\[\Xi \triangleq \frac{\sum_{i=1}^{T} |E(\mathbf{r}_i) - \hat{E}(\mathbf{r}_i)|^2}{\sum_{i=1}^{T} |E(\mathbf{r}_i)|^2}\]  

(11)

determined with a fast relevant vector machine (RVM)-based local search strategy [23] by maximizing the following likelihood function:

\[\Phi(\eta, \mathbf{z}) = \left\{ -\frac{1}{2} \left[ 2M \log 2\pi + \log |\mathbf{Q}| + (\delta^*)^{-1} \mathbf{Q} \right] \right\} \]  

(9)

starting from initial guess of the BCS noise variance, \(\eta_0\). In (9), |.| is the matrix determinant, \(\mathbf{Q} = \eta I + \Delta (\text{diag}(\mathbf{r}))^{-1} \Delta^*\), \(M\) is the number of probing locations, and \(\mathbf{I}\) is the identity matrix. Finally, the solution of (5) is computed by rearranging the entries of the BCS vector (8) into the complex-valued expansion weights as follows:

\[\hat{w} = \left\{ (\tilde{\omega}_b + j\tilde{\omega}_{b+B}) ; b = 1, \ldots, B \right\} \]  

(10)

\(j = (-1)^{1/2}\) being the imaginary unit, while the corresponding estimated field radiated by the AUT at the prediction locations \(T\) is retrieved by inputting (10) into (1).

III. NUMERICAL VALIDATION

The objective of this section is twofold. On the one hand, representative results from an exhaustive numerical study are reported to assess the effectiveness of the proposed BCS-based approach also in comparison with a previously published state-of-the-art CS approach based on the orthogonal matching pursuit (OMP) [20]. On the other hand, this section is aimed at providing the interested readers/users with some useful guidelines for its optimal application.

The radiators have dimensions \(f_1, f_2\) \(= (2.2 \times 10^{-1}, 3.3 \times 10^{-1})\) \([\lambda]\) and they are etched in a (\(\lambda/2\))-spaced square lattice on a dielectric substrate with relative permittivity \(\varepsilon_r = 4.7\), loss tangent \(\tan \delta = 1.4 \times 10^{-2}\), and thickness \(h = 1.9 \times 10^{-2}\) \([\lambda]\). Moreover, the feeding architecture consists of \(S = N_y = 10\) uniformly excited clusters/planks corresponding to the rows of the array, the nominal/gold excitations being set to \(\zeta_b = 1.0\) \((b = 1, \ldots, S)\).

As for the NF setup, \(\Psi_0\) is a square plane of side \(L_{\Psi_0} = 20\) \([\lambda]\) placed \(H = 7\) \([\lambda]\) above the AUT top surface. As for the probing location set \(M\), it consists of \(M = (M_x \times M_y = (5 \times 5) = 25\) positions uniformly distributed over \(\Psi_0\) with a step of \(\Delta_x = \Delta_y = 5\) \([\lambda]\) [20]. To account for mutual coupling effects, the basis \(\mathcal{A}\) has been built by modeling the AUT within the Altair FEKO FW simulation environment [25] by considering a \((\lambda/2)\)-sampled

\[
\mathcal{P}(\omega|\delta) \text{ is computed as follows [24]:}
\]

\[
\hat{\omega} = \frac{1}{\eta} \left[ \left( \frac{\mathcal{A}}{\eta} \right)^* + \text{diag} \{ \tilde{\mathbf{z}} \} \right]^{-1} \left( \frac{\mathcal{A}}{\eta} \right)^* \mathbf{z}
\]

(8)

In (8), \(\eta\) and \(\tilde{\mathbf{z}} = \{ \tilde{\omega}_b ; b = 1, \ldots, (2 \times B) \}\) are the estimated BCS noise variance and the BCS hyperparameters, respectively. They are

Fig. 3. BCS calibration \((H = 7\) \([\lambda]\), \(L_{\Psi} = L_{\Psi_0} = 20\) \([\lambda]\), \(M = 25\), \(T = 1681, N = 60, |z| = 0.45, \zeta = (\pi/3)\) \([\text{rad}]\), and \(SNR \in [20, 50]\) \([\text{dB}]\)\)—behavior of the NF integral error, \(\Xi\), as a function of the BCS parameter \(\eta_0\).

Fig. 4. Numerical validation \((H = 7\) \([\lambda]\), \(L_{\Psi} = L_{\Psi_0} = 20\) \([\lambda]\), \(M = 25\), \(T = 1681, N = 60, |z| = 0.45, \zeta = (\pi/3)\) \([\text{rad}]\), and \(SNR \in [20, 50]\) \([\text{dB}]\)\)—behavior of the NF integral error, \(\Xi\), as a function of the SNR.

Fig. 5. Numerical validation \((H = 7\) \([\lambda]\), \(L_{\Psi} = L_{\Psi_0} = 20\) \([\lambda]\), \(M = 25\), \(T = 1681, N = 60, |z| = 0.45, \zeta = (\pi/3)\) \([\text{rad}]\), and \(SNR \in [20, 50]\) \([\text{dB}]\)\)—behavior of the NF integral error, \(\Xi\), as a function of the SNR.
and $\hat{c}$ values for each the MebD guidelines [20], the excitations of each $s$ one, which are associated with nonidealities on both the magnitude $C$ and the OMP methods when processing noisy data with (a) SNR $50$ [dB], (b) $40$ [dB], (c) $30$ [dB], and (d) $20$ [dB].

For the numerical study, the antenna at hand is assumed to be an Intel Xeon CPU @ 3.5 [GHz] and 64 [GB] of RAM memory.

In Fig. 2, is equal to $\Delta_{FW} \approx 3$ [min] on a PC equipped with an Intel Xeon CPU @ 3.5 [GHz] and 64 [GB] of RAM memory. For the numerical study, the antenna at hand is assumed to be potentially affected by $C = (2 \times S) = 20$ deviations from the gold one, which are associated with nonidealities on both the magnitude $\chi_c = |\chi_c|$, $\chi_c^{\min}, \chi_c^{\max} = [0, 1]$, and $c = 1, \ldots, S$ and the phase $\angle \chi_c = \angle \chi_c^{\min}, \chi_c^{\max} = [-\pi, \pi]$, and $c = S + 1, \ldots, C$ of the excitations of each $s$th ($s = 1, \ldots, S$) subarray. According to the MebD guidelines [20], $K_c = 7$ simulations have been performed for each $c$th ($c = 1, \ldots, C$) uncertainty factor to yield a basis of $B = 40$ vectors, $Q_c = 2$ being the number of truncated singular values for each $c$th index. Accordingly, a total FW simulation time of $\Delta t \approx 420$ [min] has been required to build the overcomplete basis.

A preliminary calibration of the BCS setup has been carried out by assuming as reference an AUT affected by a partial failure on the excitation coefficient of the $s = 3$rd row (i.e., $|\chi_s| = 0.45$ and $\angle \chi_s = (\pi/3)$ [rad]). Toward this end, the initial guess of the BCS noise variance, $\eta_0$, for the RVM-based maximization of (9) has been varied within the range $10^{-7} \leq \eta_0 \leq 10$, and the value of the NF integral error (11) has been computed for different noise levels ($SNR \in [20, 50]$) [dB]). The outcomes of this analysis are summarized in Fig. 3. As expected, the optimal value of $\eta_0$ depends on the SNR (e.g., $\eta_0^{\text{opt}}|\text{SNR}=50$) $< \eta_0^{\text{opt}}|\text{SNR}=40$) $< \eta_0^{\text{opt}}|\text{SNR}=30$) $< \eta_0^{\text{opt}}|\text{SNR}=20$) $< \eta_0^{\text{opt}}|\text{SNR}=0$) since, by definition, the larger is the noise variance, the lower is the SNR. On the other hand, no a priori accurate information on the noise level is available in several practical cases; thus, an optimal tradeoff value has been chosen as $\eta_0^{\text{opt}} \doteq (\int \eta_0^{\text{opt}}|\text{SNR}d\text{SNR})/\int d\text{SNR}$ and it has been set here to $\eta_0^{\text{opt}} = 10^{-2}$ (see Fig. 3). To assess the reliability of such a calibration in general operative conditions, the normalized error map (i.e., $|\Delta \tilde{E}(r_t)| \doteq |\tilde{E}(r_t) - E(r_t)|/\max_{r_t} |E(r_t)|$) is reported in Fig. 4 (left column) for different SNRs.

It turns out that the NF estimation error is always very small and upper bounded to $\max_{r_t} |\Delta \tilde{E}(r_t)|/SNR=20=27.2$ [dB] [see Fig. 4(g)]. For comparisons, the maps yielded with the OMP-based implementation [20] are reported (see Fig. 4—right column), as well, to pictorially underline the better performance of the BCS approach that is quantitatively confirmed by the plots of the corresponding errors (see Fig. 5). As a matter of fact, the BCS is more robust to the noise, and it remarkably reduces the NF error especially in the worst case conditions [e.g., $SNR = 20$ [dB]) — (BCS)/OMP $\approx 21$ [dB], and Fig. 4(g) versus Fig. 4(h)]. Independently on the SNR, the BCS provides more sparse.
solutions than the OMP (i.e., \( \|\tilde{B}_{BCS}\|_0 < \|\tilde{B}_{OMP}\|_0 \)) and, unlike this latter, the retrieved nonnull entries of \( \tilde{B}_{BCS} \) when \( SNR > 20 \) [dB] are in correspondence with the four redundant\(^1\) basis vectors associated with the uncertainty factors affecting the actual AUT and highlighted with the vertical gray bar in Fig. 6. As a matter of fact, \( \|\tilde{B}_{BCS}\|_0 > 0 \) when \( b = 5 \) (i.e., the first column of \( \mathbf{A} \) linked to a variation of \( |\zeta_3| \), \( b = 25 \), and \( b = 26 \) (i.e., the basis vectors corresponding to \( \zeta_3 \)), as shown in Fig. 6(a)-(c). This points out that the BCS does not only improve the estimation accuracy of the NF distribution in \( T \) of the OMP, but it also generally provides the information on which defect/anomaly is deviating the AUT pattern from the ideal one.

To investigate the impact of the NF prediction on the FF characterization of the AUT, let us analyze the error maps of the BCS solution technique and its superior capability of mitigating the truncation error. Toward this end, the side support, guidance, and help.

\(^{1}\)Since the basis set is overcomplete, an NF field prediction needs at least one component for each uncertainty factor. In this example, there are two nonnull entries for each of the 3 = 2 uncertainty factors. Therefore, one entry for each factor is mandatory.

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REFERENCES


