Influence of joint rotational stiffness on the design optimization of grid shells

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Abstract. Grid shell structures are spreading in the last two decades thanks to their capacity to cover large span with light solutions. The peculiarity of these structures is to combine aesthetic/functional characteristics with optimal structural performances, which are completely merged since their shape and structural elements are the same thing. In this context, structural optimization techniques are increasingly adopted to support architects and structural engineers in the design of competitive constructions, usually in terms of structural weight, stiffness properties, cost and compliance specific requirements. In case of grid-shells, the optimal design process is strongly related to their susceptibility to global buckling phenomena, which are mainly related to the global stiffness of the structures. The main aspects that affect the susceptibility to global buckling are the rotational stiffness of joints, the boundary conditions and the presence of imperfections. In this framework, the paper presents a design optimization strategy that aims to minimize the weight by taking into accounts the rotational stiffness of the joints. In particular, the proposed approach pursues the objective to minimize the weight of the structure by specifically taking into account different levels of the rotational stiffness of the joints, while imposing specific structural requirements. The approach has been applied to case studies characterized by different boundary conditions, also considering the presence of imperfections. The results of the proposed design strategy highlight the beneficial effect of considering the rotational stiffness of the joints as a further parameter of the optimization, since it particularly influences the susceptibility of the grid shell to global buckling phenomena and the sensitivity to imperfections.

INTRODUCTION

Grid shells are fascinating examples of structures able to combine aesthetic/functional aspects and structural performances, thanks to their capacity to cover large span with light solutions, by exploiting the inherent strength of a double curvature shell, also if composed by a discretized mesh. The widespread adoption and the efficiency of this structural system are confirmed by several examples of grid shells realized in different parts of the world, among which the British Museum canopy [1], the Smithsonian Museum canopy [2], the Palacio de Comunicaciones canopy [3], the canopy of the lobby of DZ Bank, the Hippo House at the Berlin Zoo [4], the Orangery grid shell at Chiddingstone Castle, the Mannheim Multihalle [1], [5].

The peculiarity of the grid shells is certainly their lightness, which therefore implicates their susceptibility to global buckling phenomena. In this framework, an important role is played by the rotational stiffness of joints that could particularly affects the global stability of the grid shells [6], as testified by the collapse of a grid shell dome in Bucharest [7], caused by a design based on the hypothesis of rigid joints. In this context, Fan et al. [8] proposed a classification for grid shells properly based on the level of rotational stiffness of joints: rigid joints, characterized by high rotational stiffness; semi-rigid joints, characterized by moderate rotational stiffness; pinned joints, characterized by low rotational stiffness. Several recent literature works devoted their interest towards the characterization of the stiffness of joints for grid shells, and their effect on the structural performance. More in details, Zhang and Feng [9] investigated the cyclic behaviour of double-ring joints for grid shells, in order to define their stiffness, bearing capacity and energy dissipation; Han et al. derived the stiffness and the bearing capacity of innovative Assembled Hub joints and analysed the effect of the joint stiffness on the stability of grid shell case studies [10], [11]; Fan et al. [12] developed an experimental campaign with the aim to define moment-rotation curves of semi-rigid joints, characterized by different geometrical parameters; Ma et al. [13] investigated a new type of semi-rigid bolt-column joints constituted by a ring to which different shapes of cross-section members could be connected; Gidófalvy et al. [14] analysed the initial stiffness and the flexural resistance of an innovative joint connecting T-cross sections, by varying their geometrical features and pretension on the bolts.

Furthermore, several works focused the attention on the geometric imperfections that could modify the ideal shape during the construction process and also affect the global stability of the grid shells [15], [16]. In order to find the worst imperfection shape, a recent work proposed optimization strategies [17] that minimize the buckling factor by varying the nodal coordinates of the grid shell.

From the point of view of the structural design, optimization techniques are currently widespread in the world of both practitioners [18]–[23] and researchers [24]–[32], and the current literature also provides examples of design strategies for grid shell structures that combine optimization techniques with form finding approaches [33]–[42].

In this framework, the paper presents a design optimization strategy that aims to minimize the weight by taking into accounts the presence of semi-rigid joints. The approaches here proposed have been applied to two case studies characterized by different boundary conditions, imperfections and different shapes, in order to also investigate the effect of these on the structural weight.

CASE STUDIES

The selected case studies for the numerical simulations are two single layer grid shell structures already analysed in recent literature works [33], [36]–[39], which are characterized by a square grid composed of joints equally spaced in both directions (Figure 1).

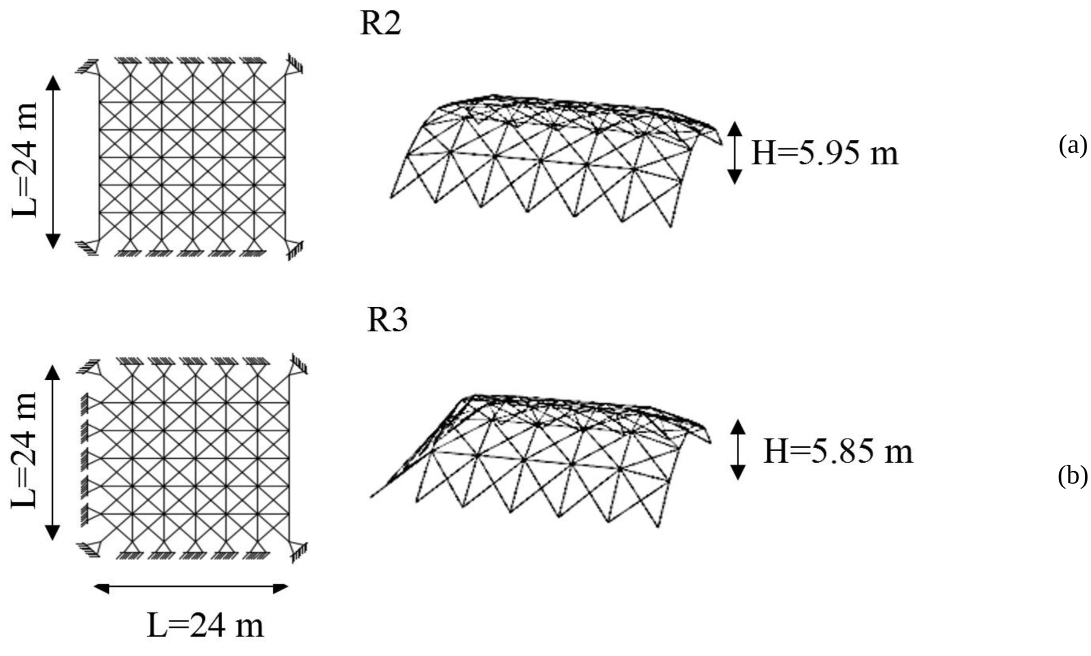

FIGURE 1. Grid shell case studies: (a) R2 - restrained on two sides; (b) R3 - restrained on three sides.

The members of the grid shells have a hollow circular cross sections made of steel S355 (yield strength: 355 MPa; Young's modulus: 200 GPa), characterized by a diameter Φ, which is a variable of the optimization process, , and a thickness *t* imposed equal to 0.09 times the diameter. Two different schemes of boundary conditions have been considered, in order to investigate their effect on the structural behaviour: in the first scheme, the grid shell is pin supported along two opposite sides (R2 - Figure 1a); in the second one the grid shell is pin supported along three sides (R3 - Figure 1b). For both the schemes, a uniform gravity load equal to 3 kN/m^2, which includes an estimate of the grid shell dead and live loads, has been considered [33].

The geometrical models have been carried out in the Grasshopper environment [43], [44], i.e. a plug-in of Rhinoceros [44] for algorithmic-aided design. The structural analyses have been performed by Karamba [45], [46], the Grasshopper plug-in for finite element analyses [43], [44].

In addition, the two grid shells have been characterized by the hanging model shape, derived by a form-finding process carried out by using the software Kangaroo [47], the Grasshopper physical engine where the Dynamic Relaxation method [48] is implemented.

Preliminary analysis: the role of joints stiffness and restraint conditions

Fan et al. [8] proposed a subdivision of joints for grid shells into three categories, on the basis of their stiffness: rigid joints; semi-rigid joints and pinned joints. In this context, a grid shell can be considered characterized by rigid joints if it is equipped by joints with a finite value of stiffness that provide a buckling factor BF larger than 90% of the BF reached in case of fully rigid joints with infinite stiffness; on the other hand, a grid shell can be considered characterized by pinned joints if the stiffness of the joints leads to a buckling factor BF lower than 25% of the BF reached in case of fully rigid joints with infinite stiffness; in the intermediate cases, the grid shell is considered equipped with semi-rigid joints [8]. With reference to the two case studies, characterized by different restraint conditions (R2 and R3 - Figure 1), the buckling factor has been evaluated by varying the joints rotational stiffness k_j, for a fixed cross-section that guarantees an adequate buckling factor also in the case of pinned joints; FIGURE 2 shows the trend of the ratio between the BF and BF_{rigid} (evaluated for fully rigid joints) with respect to the ratio between the rotational stiffness of the joints k_j and the average flexural stiffness of the elements $k_{e,av}$:

$$k_{e,av} = \frac{EI}{L_{av}} \quad (1)$$

where E is the elastic modulus of the steel, I is the modulus of inertia of the cross-sections, and L_{av} is the average length of the members.

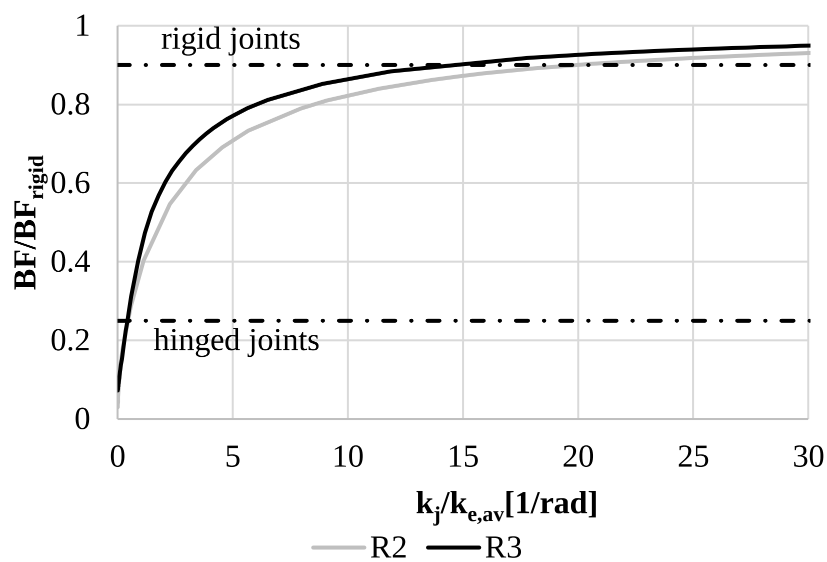

FIGURE 2. Ratio between the actual Buckling Factor BF and the Buckling Factor in case of fully rigid joints BF_{rigid} VS ratio between joint rotational stiffness k_j and element flexural stiffness $k_{e,av}$.

FIGURE 2 highlights that the R2 case behaves as a grid shell composed by rigid joints for $k_j/k_{e,av}$ larger than 20, while the R3 case for $k_j/k_{e,av}$ larger than 15; on the other hand, both cases behave as a grid shell with pinned ends for $k_j/k_{e,av}$ lower than 0.25. These results show that, other than the rotational stiffness of the joints, also the boundary conditions play a fundamental role on the global behaviour of grid shells. Moreover, an adequate categorization of the behaviour of the joints cannot be performed a priori, since it also depends on the boundary conditions. In this framework, it is useful to define design strategies able to assign an adequate stiffness to the joints by considering different external conditions, which is properly the aim of the paper.

PROPOSED APPROACH

The optimization approach here proposed, denoted in the following as joint stiffness approach, consists of a sizing optimization process finalized to find the lightest solution of grid shells by specifically taking into account the rotational stiffness of joints. In detail:

For each	$k_j/k_{e,av}$	
Minimize	W	
Subjected to	$BF \geq BF_{lim}$	(2)
	$U_{max} \leq U_{lim}$	
	$D_{max} \leq D_{lim}$	
	$\Phi_{lb} \leq \Phi \leq \Phi_{ub}$	
Variables	Φ	

where: W is the structural weight of the grid shell, BF and BF_{lim} are the actual and the limit value of the Buckling Factor, respectively; U_{max} and U_{lim} are the maximum and the limit value of the Utilization ratio (i.e. a demand to capacity ratio that also includes local buckling), respectively; D_{max} and D_{lim} are the maximum and the limit displacement, respectively; Φ_{lb} and Φ_{ub} are the lower and upper bound values of the diameter Φ, respectively. The limit value of the buckling factor BF_{lim} has been set equal to 3; the limit value of the Utilization ratio U_{lim} is imposed equal to 1; the limit value of the displacement D_{max} is 0.096 m, i.e. the maximum span divided by 250; Φ_{lb} and Φ_{ub} are equal to 5 cm and 20 cm, respectively.

The optimization process has been applied to the perfect shape, i.e. the one deduced from the hanging model shape, and to an imperfect shape that takes into account of geometrical imperfections, which can occur during the construction process. In particular, the imperfect shape has been obtained by an optimization process that minimizes the BF by varying the joints position in the range ±0.05m (L/500) with respect to the perfect shape, as proposed by Tomei et al. [17]. The range of variation adopted for the joints coordinates has been chosen according to the recommendations reported in previous literature works analyzing the amplitude of imperfections [6], [49].

The optimization problem is solved by a mono-objective genetic algorithm, which is implemented in the component Galapagos of Grasshopper [43], [50]. In general, genetic algorithms are based on the concept of natural selection: indeed, starting from a population of individuals generated in a random way, where each individual is a potential solution, the algorithm selects the best individuals (the parents) to produce different generations of children, until reaching a population of individuals which contains the optimal solution [51]. In particular, the main steps of Galapagos are:

- creation of the first generation of random individuals;
- evaluation of the Objective Function, i.e. the quantity to minimize, for each individual of the current generation;
- selection of the best individuals (the parents) to survive, to couple and to mutate in order to create the children of the next generation.

Then, the optimization process ends when the maximum number of generations is reached, there is no progress for a specified number of generations, or a specific fitness value is achieved.

In reference to the proposed approach, the joint stiffness approach aims to provide the minimum weight design for grid shells, by varying the rotational stiffness of the joints and the cross-sections to assign to the elements.

Errore. L'origine riferimento non è stata trovata. shows the minimum weight W obtained from the optimization process by varying the ratio $k_j/k_{e,av}$ for both R2 and R3 grid shells, normalized with respect to the weight in case of hinged joints W_{hinge}: in particular, the minimum weight highly decreases by increasing the ratio $k_j/k_{e,av}$ from zero, i.e. the hinged joint configuration, to 5, while the reduction is less pronounced from $k_j/k_{e,av}$ equal to 5 until approaching the weight obtained for rigid joints. Furthermore, the imperfections lead to an increase of weight of 2.9 and 1.7 times in the case of hinged joints, respectively for R2 and R3 case; then, this difference rapidly reduces by increasing the ratio $k_j/k_{e,av}$. This indicates that a small increase of the joint rotational stiffness greatly reduces the susceptibility of the grid shell to imperfections. Moreover, the comparison of the results in terms of weight for R2 and R3 suggests that the weights obtained for R3 are ever lower than those obtained for R2, reaching a reduction of 47% for the perfect shape and 88% for the imperfect shape. These results highlight the influence of boundary conditions on the sensitivity to global buckling phenomena: indeed, the lower the number of restrained sides, the greater the optimal weight. FIGURE 4 plots the maximum value of the Utilization ratio U_{max} obtained for each optimal solution; in particular, by increasing the value of $k_j/k_{e,av}$, also U_{max} increases, moving toward solutions that exploit at best the cross-section from a structural point of view. The results in terms of maximum displacements D_{max} are not reported, since these are ever much lower that the limit one D_{lim}.

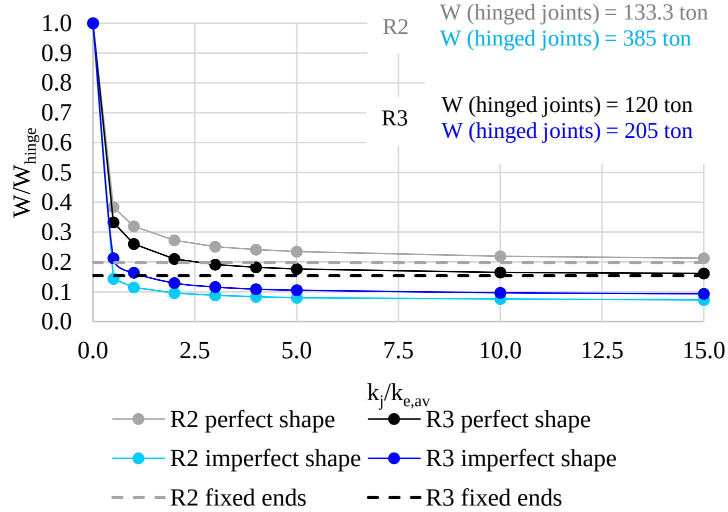

FIGURE 3. Weight W normalized to the weight in case of hinged joints W_{hinge} VS ratio between joint rotational stiffness k_j and element flexural stiffness $k_{e,av}$.

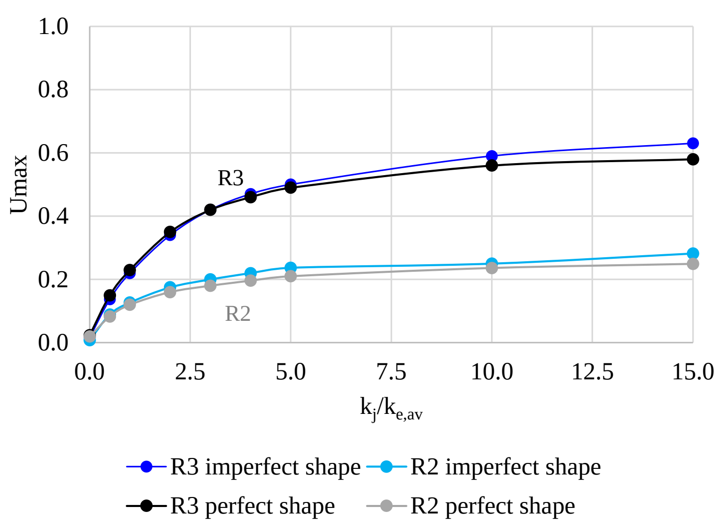

FIGURE 4. Maximum Utilization U_{max} VS ratio between joint rotational stiffness k_j and element flexural stiffness $k_{e,av}$.

The role of the shape

One of the peculiar aspects of grid shells, is given by the possibility of giving them particular and fascinating shapes, which therefore affect both the architectural features and the structural aspects. In order to investigate the structural meaning of the shape, the joint stiffness approach has been applied to grid shell R3 with the shape derived by a form-finding process, but characterized by a maximum height reduced of 50% (FIGURE 5 – shape 2) with respect to the solutions already described in FIGURE 1 (Figure 7 - shape 1).

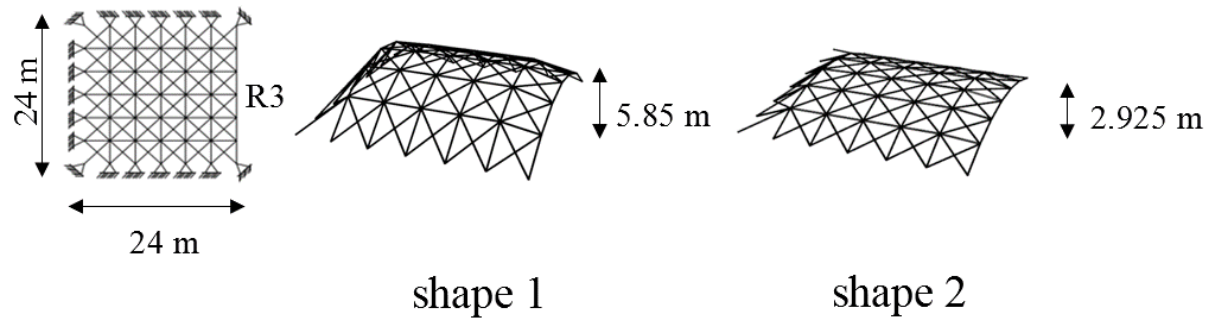

FIGURE 5. Grid shell case studies: shape 1 vs shape 2.

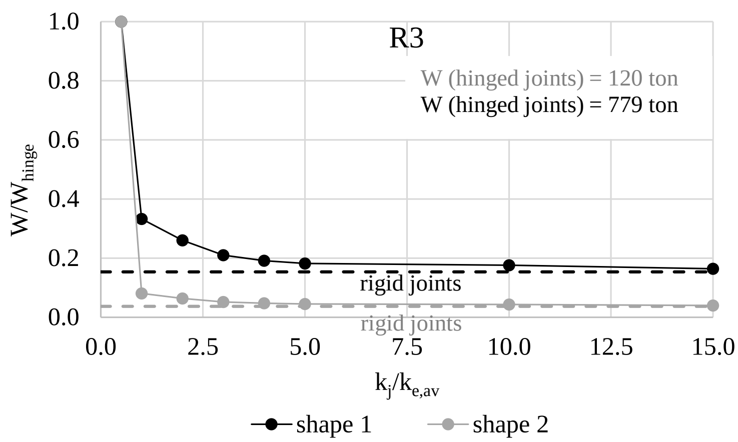

FIGURE 6. Optimization problem results for grid shell R3: Weight W normalized to Weight in case of hinged joints W_{hinge} VS ratio between joint rotational stiffness k_j and element flexural stiffness $k_{e,av}$ for shape 1 and shape 2.

FIGURE 6 shows the results in terms of W/W_{hinge} VS $k_j/k_{e,av}$ for the two shapes conferred to grid shells. Although the trend of the two curves is similar, shapes 2 provides larger values of the weight; this difference in terms of weight reduces by increasing the ratio $k_j/k_{e,av}$, going from about 650% in case of hinged joints to150% in case of rigid joints. This is due to the fact that the shape 2 is characterized by a higher curvature than shape 1 implying that the grid shell tends to behave as a "plate", meaning that the grid elements work in bending rather than compression; on the other hand, shape 1 is characterized by a lower curvature than shape 2, which led the grid shell toward a "dome" behaviour, where the grid elements work predominantly in compression. In the light of this discussion, it is clear that, since the bending stiffness of a slender beam is much less than its axial stiffness, if the elements are forced to work in bending, as in the case of a "plate" behavior, higher cross-section sizes are needed This result emphasizes the fundamental structural meaning of the shape, and its role in conceiving minimum weight solutions.

CONCLUSIONS

The structural weight of grid shells is strongly connected to their susceptibility to global buckling phenomena, which is often the governing design criterion. This susceptibility is strictly related to the global stiffness of the structures, which is conferred by the stiffness of the joints, the boundary conditions, the presence of imperfections. In this framework, the paper presents a design optimization strategy that specifically takes into accounts the presence of semi-rigid joints in order to find light solutions safe from global buckling. The proposed approach, called joint stiffness approach, considers the grid shell composed by semi-rigid joints characterized by a certain value of rotational stiffness, and provides for minimizing the structural weight by imposing constraint conditions on the buckling factor on the maximum level of utilization ratio and on the maximum nodal displacement. In order to analyse the influence of boundary conditions and imperfections on the buckling factor, the approach has been applied to case studies characterized by a different number of the restrained sides, and also considering both perfect and imperfect shapes. The obtained results provide the following considerations:

- the results of the proposed optimization processes highlight the beneficial effect of a finite value of the joint rotational stiffness, also if small, in the susceptibility of the grid shell to global buckling phenomena; in particular, it leads to an increase of the buckling factor with respect to the configuration of hinged joints (zero stiffness), which allows to use smaller cross-sections, and, hence, reduce the structural weight;
- a finite value of the joint rotational stiffness, also if small, reduces the susceptibility of the buckling factor to geometrical imperfections;
- the boundary conditions affect the sensitivity of the grid shell to global buckling phenomena: the lower the number of restrained sides, the higher the structural weight necessary to assure an adequate stiffness against buckling phenomena;
- the shape of grid shells assumes a fundamental structural meaning; as the curvature decreases, the grid shell tends to behave like a "dome", which leads the structural elements to work in compression rather than bending, guaranteeing smaller cross-sections compared to a grid shell characterized by higher curvature, which tends to a plate behavior; this leads the last grid shell to be heavier than the first one, with the same stiffness of the node. Questa però deve essere correlate sempre alla rigidezza dei nodi

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