Geotechnical-structural engineering for the preservation of Ninfeo Ponari in Roman Casinum

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ABSTRACT: The Ninfeo Ponari is the unhearted part of a domus built around the first century BC in the Roman city of Casinum on the lower slope of the Monte Cassino. Within a bigger archaeological complex present in this area the *nymphaeum* witnesses the historical-cultural relevance of this territory during the Roman age. Thus recovery, preservation and opening to public are considered a priority by local and national institutions. The structural integrity is threatened by the weight of debris fallen along the slope, that have totally overlaid the building. Different competences are involved to detect the composition and mechanical properties of the structural elements, monitor the geometrical-structural layout, investigate the subsoil conditions. A three-dimensional Finite Elements Model has been implemented, it serves to assess the present stability conditions and future recovery actions through the simulation of the soil-structure interaction. The analysis has required reconstructing the monument history through the interpretation of artifacts combined with the knowledge brought by archaeologists of the ancient construction techniques. The paper presents an overview of the ongoing activities, summarizes the temporary conclusion and pave the way for the remediation strategy.

INTRODUCTION

The cultural legacy of Roman civilization has left various traces over the Italian territory. Together with popular monuments that capture the interest of historians and tourists, there are plenty other examples of unknown but not less important evidence that deserve the same attention. Frequently they are in precarious conditions due to the natural and anthropic events occurred along with centuries and thus require an action for their recovery, conservation, reopening to the public use and valorization. The present example concerns the Ninfeo Ponari, part of a rich domus built around the first century BC in the Roman city of Casinum (Valenti, 1992). Although included in some repertoires on ancient nymphaea (Valenti, 1992), this monument is still not fully explored, being forgotten and hidden for years and now presenting structures in a precarious state. In order to restore the monument with an efficient intervention that respects its historical and cultural value, in recent years several explorative campaigns have been undertaken to study the manufact from the historical perspective (Valenti, 1995; Caponero et al., 2020), to assess its stability conditions and figure out effective strategies.

The competences of archeologists, experts of Roman history and tradition, in understanding the function of the different rooms, their connection with the surrounding context and the construction practice of the building, has been merged with those of technicians, expert of geotechnics and structures to back-analyze past soil-structure interaction throughout the monument life, quantify the present stress state on the different component in comparison with the conservation state of materials and assess the stability of the structural elements, walls, vaults, buttresses.

A fundamental preliminary step of the analysis has consisted in a complete, deep and detailed topographic survey of the different parts of the *domus*, carried out with up-to-date investigation tools. The results of the above analyses are summarized in the present paper focusing more particularly on the geotechnical-structural perspective. With this aim a FEM calculation has been implemented to simulate the soil-structure interaction from the construction time to date.

1 THE NYMPHAEUM

The Ninfeo Ponari is located on the lower slope of Montecassino hill, along the road linking the modern city of Cassino to the famous Benedictine abbey (Fig.1) (Valenti, 2010). This area hosts the old Roman Casinum, as witnessed by several archaeological remains that include a tomb, a theatre and an amphitheatre. The territory was conquered by the Romans around 313 B.C. and remained faithful to Rome. After a period of Civil Wars in the 1st cen-tury B.C., Casinum experienced a period of prosperity during which numerous public monuments were built. It declined due to a series of barbarian attacks between 400 and 500 B.C. (Fig.2.a). The building is part of the residential sector of a rich Roman domus, probably built around the middle of the 1st century. B.C. and restored during the 1st century A.D. (Neuerburg, 1965; Valenti, 1992) It was probably a summer dining complex (“coenatio aestiva”), enriched with fountains (hence the name “nymphaeum”). (Betori et al. 2009)

The walls, built with masonry and covered with plaster, form two contiguous and communicating rectangular spaces: a room covered with a barrel vault and provided with three niches on each side or front wall, 4.6 m wide, 7.5 m long and 4.6 m high in the top of the vault; an open atrium with a basin (impluvium) in the center, 7.5 m wide, 6.4 m long and surrounded by walls about 3.50 m high (Fig.2.b). (Carettoni, 1940; Ghini, 1995).

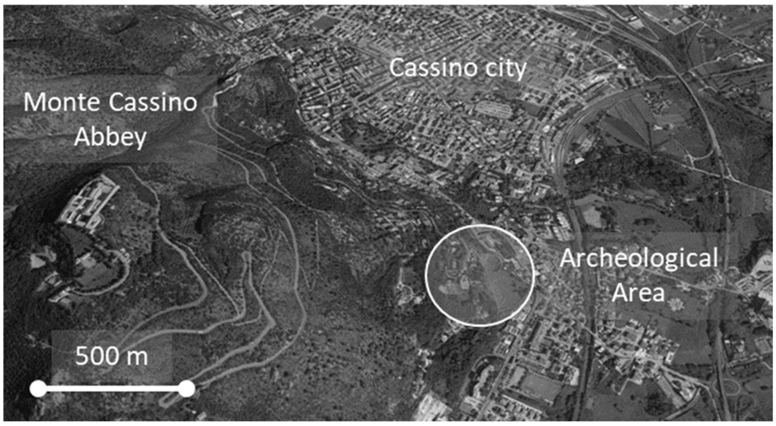

Figure 1. Geographical position of the Ninfeo Ponari

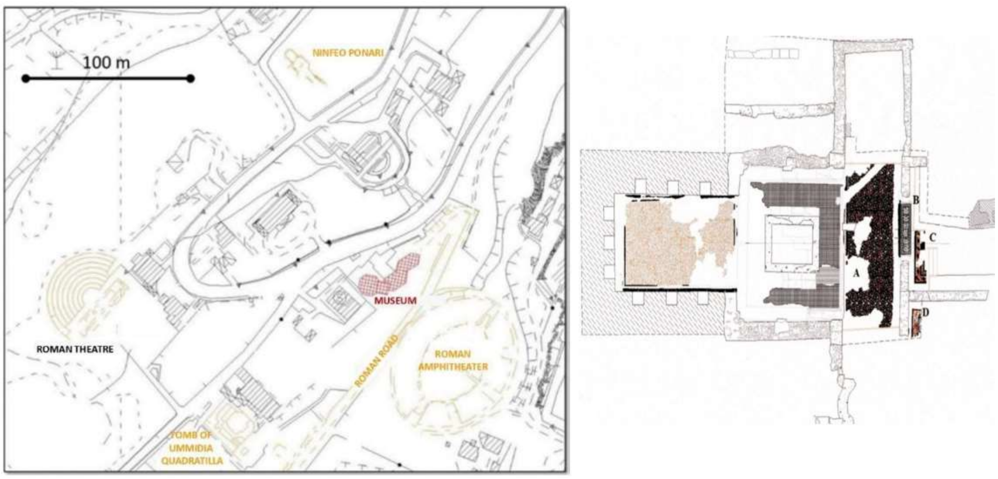

Figure 2. Layout of the archaeological area (a), plan view of the nymphaeum (b).

In the covered portion, two distinct wall decorations can be recognized: the oldest is made up of a "rustic mosaic" with shells, Egyptian blue tiles and colored glass of geometric shapes: a plaster painted with imitation of polychrome marble slabs covers the former one. Also the walls of the atrium are covered with painted plasters presenting a red base band, animated by plants and beasts and decorated with gilded bronze vases. The floors are decorated with mosaics containing polychrome marble inserts on a white or black back matrix, with concentric or meander perspective borders. (Valenti 1995, Valenti 1992)

Due to its similarity with other examples and the presence of shells and wall decorations, the building was classified as a nymphaeum, given its cave structure. It has been reasonably assumed that the nymphaeum, intended to host water games, was part of a luxurious private residence owned by one of the most important families of the ancient Casinum, the gens Ummidia. (Betori et al. 2009). Until the end of last century, the domus was left abandoned, being almost filled with earth and rubbles (Fig.3.a). A first excavation, accomplished between 1998 and 2001 (Betori et. al 2009), brought the structure to light and revealed a rich colored decoration (Fig.3.b). The presence of debris around the walls and on the vault has determined for long time, and still determines, a meaningful overload on the masonry structures. Neither the inner excavation can be assumed to relief this state. In fact, the contact of the walls with the ground continuously determines infiltration of groundwater that leaks through the masonry and contributes to deteriorate the status of mortar joints and plaster forming musk and vegetation that cover the wall surfaces (Fig.3.c).

Figure 3. Picture of the Ninfeo Ponari before (a) and after excavation (b), example of painted wall with musk and vegetation (c) and of pavement (d).

2 THE NYMPHAEUM

Different topographic surveys have been undertaken in the last years to analyze the present status of the *nymphaeum* walls and vault, monitor possible kinematic evolution and warn against ongoing deformation. In details, a photogrammetric survey has been performed to reconstruct the whole geometrical pattern of the inner surfaces implementing the “Structure from Motion” technique (Caponero et. al, 2020). The method consists in automatically collimating and recognizing common markers from different photographs. In the presents case, 703 photographs were taken by a 24 MPX camera mounted on a telescopic monopod and processed with a software (Photoscan Pro v. 1.2.6) on the ENEA CRESCO (Computational Research Center for Complex Systems, https://ict.enea.it/cresco/) platform.

A 3D model was thus created containing a cloud of about 55 million dots representative of the position and color of the surface. This output provides information on the layout geometry highlighting the possible presence of wall discontinuities, e.g. deterioration, cracks, vegetation etc. At the same time, a laser scanner survey was performed with the Leica BLK 360. This survey has herein been used to provide a reference for the above-described photogrammetry in order to scale it as shown in Fig4.a, for the covered nymphaeum portion (tablinum).

The 3D laser scanning model has also been interpreted to figure out a quantitative description of the deformation taking place on the walls. In the example of Fig.4.b, the deviation of wall inner surfaces from ideal vertical planes anchored to the back and lower edges of the walls, these latter supposed as fixed considering their proximity to rigid boundaries, is reported with a shaded plot. Assuming that walls were built approximately vertical, the present shape shows a maximum inner convergence of about 5-6 mm on the upper-outer corner. This aspect is possibly due to the thrust of the outer soil and will be explored in the next section with a numerical analysis.

Finally, several topographic surveys were performed with a total station (Leica Nova TM50 robotic), having an accuracy on angles of 0.5" and on the distance of 2 mm, on 34 spherical targets distributed on the nymphaeum walls and vault. Measurement was periodically repeated five times over a time interval of about one and half year (from August 2019 to March 2021) to detect a possible evolution of the wall movements. The unchanged position of the markers within this time suggests that the nymphaeum structure is in relatively stable equilibrium.

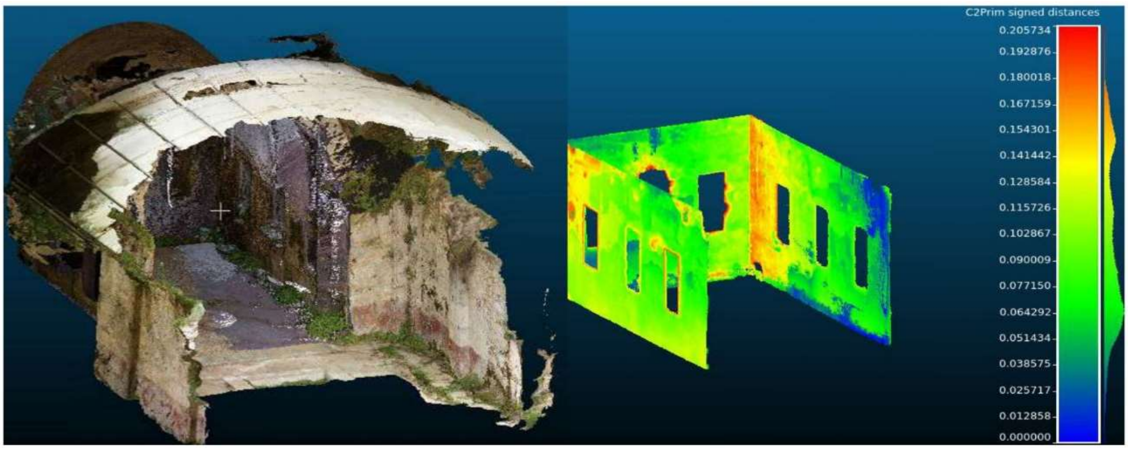

Figure 4. Geometrical three-dimensional model of the nymphaeum obtained by the combination of photogrammetry and laser scanning (a) and pattern of walls deviation from vertical reference planes (b).

3 SOIL-STRUCTURE INTERACTION

The analysis of the mechanical interaction between soil and masonry structure in these cases has different functions. It serves primarily to appreciate the present safety conditions of the monument, interpreting the role of natural and anthropic events occurred throughout its life. Next, accurate models enable to foresee the effects of possible future actions addressing remediation to effective and conservative solutions. This issue, relevant for any intervention on existing structures, becomes fundamental for delicate context such as historic sites. Finally, the model represents a benchmark for the structural monitoring as it provides a key for interpreting mechanisms.

On the other hand, when dealing with systems susceptible of irreversible phenomena, it is necessary to study stress-strain interaction not only in the present state, but also considering previous conditions that could have modified the mechanical response of materials. Reconstructing composition and history of the Ninfeo Ponari was not straightforward and implied non-invasive tests coupled with the knowledge of historical factors brought by archeologists.

Considering the geometrical layout of the manufact depicted in Figs.1 and 2, a three-dimensional finite element model has been implemented in the computer code (Plaxis 3D Ultimate CONNECT Edition 21.01) considering the *nymphaeum* with a soil portion around it. Stratigraphical and mechanical characterization of the subsoil have been accomplished with ad hoc geotechnical investigation. However, as shown later, even the reconstruction of subsoil and structure properties required the fundamental contribution of archeologists.

3.1 *Geometrical and stratigraphic model*

The geometrical set up implemented in the FEM analysis, with the identification of the different subsoil layers and the position and dimension of the manufact is accomplished combining geologic, historiographic and archaeological sources with the results of geotechnical investigation.From the geological viewpoint, the *Nymphaeum* is located on the NW-SE inclined slopes of the orographic systems including the Latium Antiappennines and the Latium-Abrutii Apennines. The deeper strata consists mainly of Mesozoic limestones displaced with normal and reverse fault systems from the Apennine tectonic action. The investigation carried out for the project consists of two continuous boreholes, one (S3) performed upstream of the artifact reaching a depth of 16m, the other (S2) downstream reaching a depth of 30 m and hosting a down hole test (Fig.5.a and b).

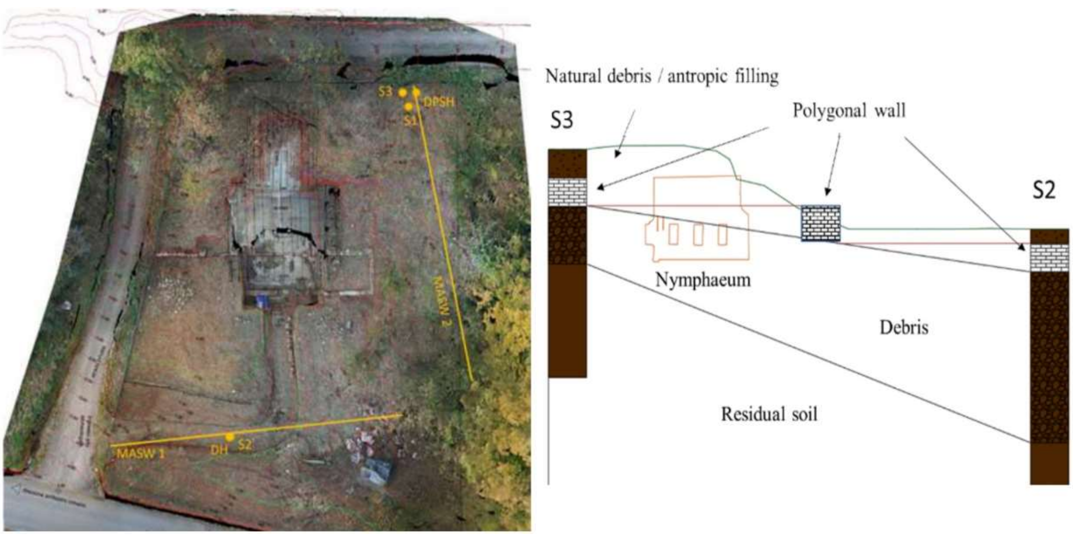

Figure 5. Plan view (a) and stratigraphic layout (b) of the Nymphaeum

The sediments emerging in the surrounding area consist of debris produced from the alteration of upstream carbonates and from Miocene clays/marly clays. Gravel and sand of calcareous nature lay often immersed in a silty or clayey matrix. Below this layer there is a base formation of residual soils (red soil) produced by the dissolution of calcareous rock.From the observation of unnatural white calcareous stones in the two boreholes and the presence of massive blocks near the *nymphaeum* (Fig.6), an arrangement in the form of terraces retained by polygonal walls was presumably created in the area at the time of *nymphaeum* erection. This assumption is confirmed by archeological studies which confirm the frequent use of the polygonal walls in the Casinum area. (Valenti, 2012) This polygonal wall, probably from the second century BC, is similar to others present in Latium, e.g. in Ferentino located 50 km to the north. The terrace of the *Nymphaeum*, discovered in the 70s, was probably partially dismantled for the construction of the *Nymphaeum*. With time, the whole area, including the interior of the *nymphaeum*, was filled with rubble mixed with debris fallen from the upper hill.

Being not possible to drill the masonry elements or to excavate the external part of the *nymphaeum*, the thicknesses of walls and vault have been assumed (Fig.7.a) considering the units typically adopted in Roman constructions: the walls have thus been assumed with a thickness of about 90 cm (one feet equal to 29.65 cm), while the vault has been considered to have the same thickness measured externally throughout its length. The model includes an initial removal from the terraced slope of a trench and its replacement with the *nymphaeum* walls (Fig.7.b and c). This assumption, suggested by archaeologists, is supported by the evidence that plaster is applied directly on the unrefined inner masonry surface.

A two-phase subdivision in the trench filing could explain the fissure now observed at middle height. The roof consisting of a masonry vault has been possibly added in a second moment. According to this sequence, the inner room has been created in the model removing soil before adding the upper vault. The subsequent submersion shown in Fig.7.d represents the filling by debris occurred with time, while the last scheme (Fig.7.e) reports the present situation. All elements of the model have been discretized with tetrahedral clusters, each having 10 calculation nodes. Global coarseness has been set as medium (re=1.00; le=2.5m) to optimize calculation effort, but a refinement (re=0.5; le=1.2m) has been accomplished near the structure to enhance the accuracy of results.

Figure 6. Picture of the polygonal wall (c). The geometrical model implemented in the analysis evolves as described in Figure 7 (from b to e).

3.2 *Structural model*

From the few parts of the wall not covered by plaster it is seen that the *nymphaeum* masonry is composed of irregularly shaped cobbles of calcareous stone, about 20 cm coarse, jointed with white mortar; rarely sparse fragments of brownish travertine limestone and isolated fragments of reddish brick are found (Valenti, 1992). Along the vault, the mortar joints are radially oriented (see Fig.2) possibly due to the use of ribs during construction and to improve the adsorption of circumferential stresses. The Jointed Rock model has been used to simulate the non-linear stress-strain response of the masonry elements. Conceived for fractured rock masses, it adopts a cross-anisotropic elastic-perfectly plastic response to simulate the pre-failure response of compact elements and a plastic model to simulate sliding along the joints oriented in preferential direction. For the present study, the elastic model has been simplified considering the masonry fabric as isotropic and assigning three elastic parameters, namely Young's and shear moduli and Poisson's coefficient. The values of these parameters have been attributed following the standard (Regione Umbria, 2003; Borri e De Maria, 2009), that classifies material with a quality index computed based on eight factors characterizing the geometrical and mechanical properties of the masonry.

A combination of these factors identifies a quality category to which typical ranges of mechanical parameters are associated. Considering the walls formed of irregularly shaped cobbles, joints not distributed with horizontal rows and vertical staggering, mortar of intermediate quality and stones only slightly degraded, the walls have been attributed to an intermediate category. The assumed elastic parameters ($E=1077.2\ N/mm^2$, $\nu=0.2$, $G=364.6\ N/mm^2$) are thus the mean values of the ranges suggested for this category. The joints orientation has been assumed as horizontal for the vertical walls, radial for the vaulted roof. Plasticity along the joint planes, simulated with a Mohr-Coulomb failure criterion, is characterized by a friction angle $\phi=30°$, a cohesion $c=10$ kPa, and nil tension cut-off, looking also at former similar studies (e.g. De Felice et al. 2015).

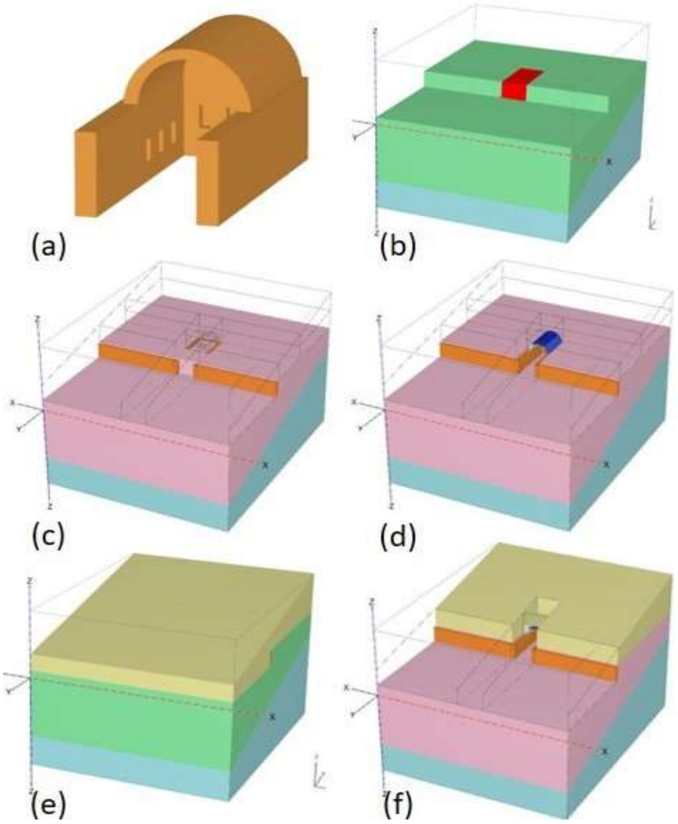

Figure 7. Geometrical model of the nymphaeum (a), layout adopted of the numerical analysis in the different cal-culation phases (b). before construction of the nymphaeum; (c). excavation and filling of the trench; (d). after construction; (e). submerged with debris; (f)

3.3 *Geotechnical model*

The geotechnical characterization of the subsoil has been carried out considering the results of a downhole test (ASTM D7400M, 2019) performed in the borehole S2 (Fig.8.a), of Dynamic Probe Super Heavy (DPSH) tests (ASTM D6951-03) performed near the boreholes S3 (Fig. 8.b) and a series of laboratory tests performed on samples taken on the lower strata in borehole S2 (Fig. 9). The downhole test confirms the presence of three layers from top to bottom with shear wave velocities equal to respectively 200, 480 and 540 m/s (Fig.8.a). The slight difference between the properties of the lower two layers is confirmed by the number of DPSH blow counts (Fig.8.b), transformed into equivalent NSPT (standard) with the LaCroix & Horn (1973) relation, then correlated to the relative density (Skempton, 1986). The consolidated drained triaxial tests (TXCD) performed on undisturbed samples cored on the lower layers (5 and 8 m depth in borehole S3) show the mean failure envelope given in Fig.9. From the summary of the above results, the subsoil has been described with a linear elastic, perfectly plastic Mohr-Coulomb model, assigning the parameters summarized in Table 1. The water table was not intercepted in the performed boreholes and thus it was neglected in the analysis.

Table 1. Parameters of theMohr-Coulomb model for the subsoil layers of Fig.1.

Soil	γ_{nat} (kN/m3)	E (GPa)	ν	c' (kPa)	ϕ' (°)	ψ (°)
Top fill	18	0.2	0.4		28	0
Debris	18	1.25	0.4	35	25	0
Residual	19	1.7	0.4	35	25	0

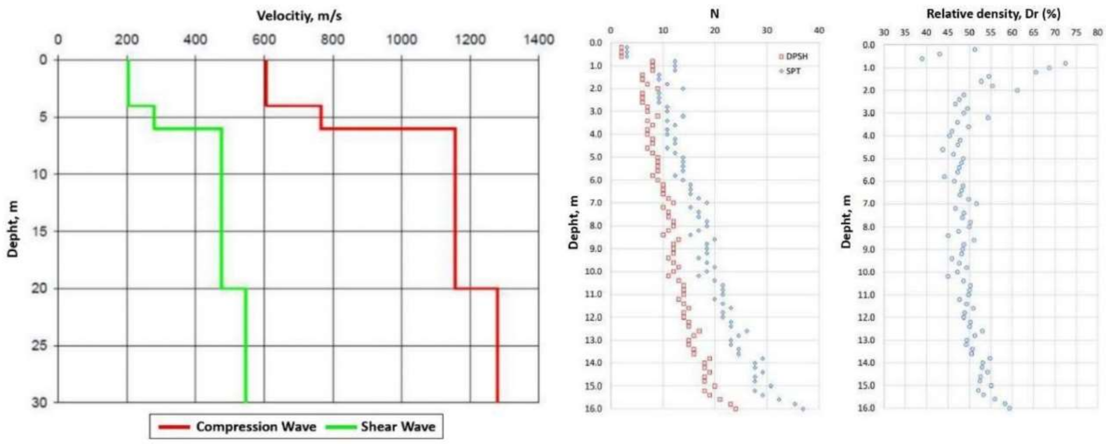

Figure 8. Shear and Compression wave velocity from downhole performed in the borehole S2 (a); DPSH performed in borehole S. (b)

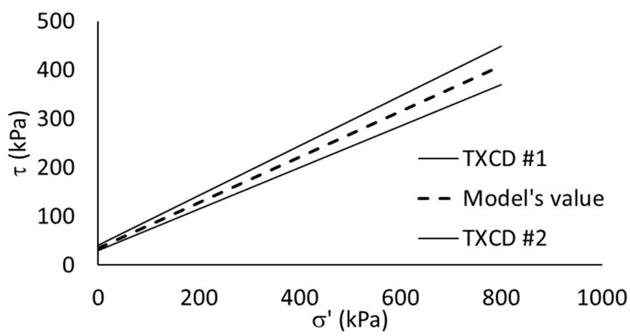

Figure 9. Triaxial test failure envelope form consolidated drained triaxial tests on samples cored from S2 in the deepest layer.

3.1 *FEM calculation results*

Calculation has been accomplished for each phase of the structure's life schematized in Fig.7, but the herein reported results refer for brevity to the present state only, considering this as the starting condition for future actions. From the deformation pattern of the *nymphaeum*, with displacements amplified one hundred times in Fig.10.a, it is readily seen that lateral soil thrust has a predominant role. In fact, the side walls tend to converge toward the inner space with displacements reaching a maximum of 6-7 mm (Fig.10.b). Apart from being quantitatively consistent with the digital survey is shown in Fig.4.b, this result is explicative of the soil/structure interaction. Convergence is maximum in the intermediate section of the wall, while movements at the borders are constrained by the end wall and base slab that react with normal stresses. Indeed, this deformation cumulates the effects of the whole *nymphaeum*'s history, from its construction to date. It begins with the creation of the rooms, i.e. after filling the trenches with masonry and removal of soil, but grows with to the subsequent covering of the area by debris fallen from the hill.

Interestingly, the debris overcharge on the side of the nymphaeum and on the upper vault produces opposite effects on the walls. In fact, while the former is detrimental for the wall stability as it increases the horizontal thrust on the walls, the soil on the vault has a stabilizing action both for the increase of vertical stresses on the walls, given by the heavier weight transferred from the vault, and for the partial horizontal constrain given by the vault presence on the walls. This issue must be carefully analyzed when considering remediation of the monument. In fact, the excavation of debris around and above the *nymphaeum* cannot proceed randomly but must be planned considering each time the stress modification induced on the structure by the soil removal. In fact, the elimination of the soil above the vault leads in the calculation to numerical instability and collapse. This result underlines the importance in modulating soil excavation on the *nymphaeum* sidewalls in a gradual, symmetrical way, before removing the soil on the vaulted roof.

Another fundamental output of the model is the scrutiny of plastic points in the structure, i.e. the points where the masonry reaches limit conditions in accordance with the Mohr-Coulomb failure criterion and tension cut-off inherent with the assigned jointed rock model (Fig.11). The outcome reveals a complex pattern of plasticization given by the intersection of phenomena developing along different directions. In particular, together with the above discussed horizontal thrust in the cross section, the growth of debris thickness in the uphill ground determines also a meaningful thrust in the longitudinal direction, i.e. along the symmetry axis of the *nymphaeum*.

The computed movements become higher when the analysis is performed considering the heavier unit weight that may occur after heavy rain. This phenomenon, highlighted by the horizontal alignment of plastic points at middle height of the walls and at the vault abutment. is consistent with the direct observation of damage on the *nymphaeum* (Fig.12.a). Another important effect shown by the plastic point distribution is the weakness of the walls in the open atrium (Fig.12.b).

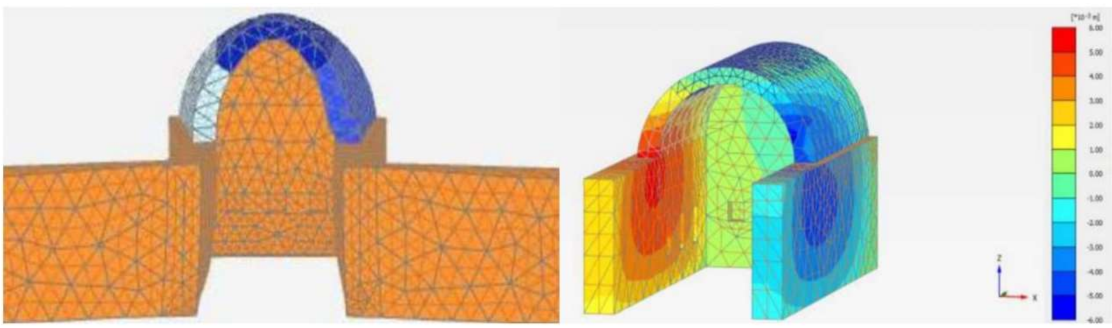

Figure 10. Deformation of the nymphaeum amplified one hundred times (a) and shaded plot of horizontal displacements (b).

At present, these walls are in a very critical state, being subjected to a significant inward tilting that have alarmed the operators encouraging to undertake local stability intervention in the last years. In conclusion, the structural response is dominated throughout the *nymphaeum* life by the interaction with the outer soil, that represents the main cause of concern, but severity increases with the weight of up-hill soil as confirmed by the increased number and extension of plastic and tension cut-offs points observed in the numerical calculation.

4 CONCLUSIONS

The topographical survey of the Ninfeo Ponari carried out at different times show that the structure is not undergoing significant movements and, at least in the last few years, that has reached a stable condition. However, the general analysis of the geographical situation, with the *nymphaeum* positioned on a slope filled with debris, highlights the need for a more careful assessment. The comparison between the shape of the inner wall surfaces, detected by means of topographical surveys, and the ideal planar conditions highlights a possible inner convergence of the side walls with maximum displacements approximately equal to 6-7 mm in the upper intermediate section. The simulation with the numerical model confirms this deformation pattern and strictly correlates it with the interaction between structure and surrounding soil. In particular, the negative role of the soil load on the *nymphaeum* walls, present from the construction times but exaggerated by the presence of debris fallen in the last centuries all over the area, jeopardizes the *nymphaeum* stability. Additionally, the water infiltrated in the ground after rainfalls determines heavier static conditions and contributes to the deterioration of frescoes on the walls. Remediation must thus aim to mitigate this interaction. The calculation has also highlighted the need for paying extreme attention to the balance given by the ground placed on the side and vault of the *nymphaeum* as a random removal might cause serious consequences.

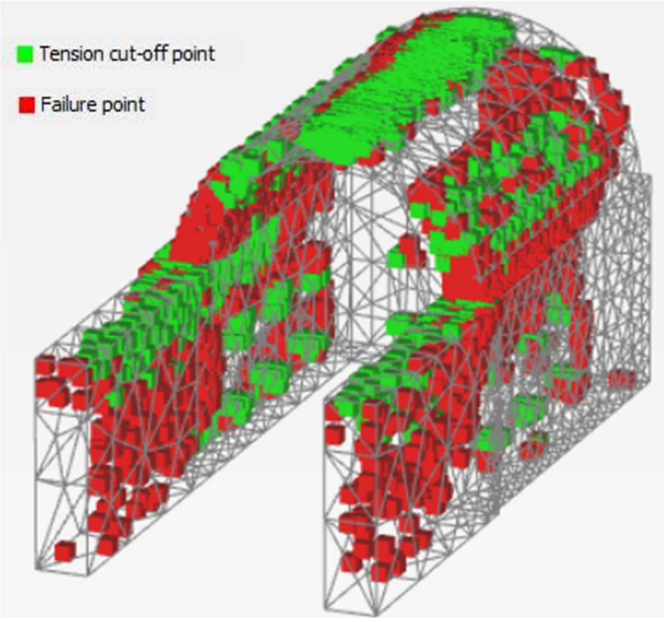

Figure 11.Plastic points from the model

Figure 12. Cracks along the vaulted room (a) and rotated walls in the outer space (b) of the nymphaeum.

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