

KINEMATIC AND CURVATURE ANALYSIS OF THE MCPHERSON EIGHT-BAR SUSPENSION SYSTEM

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ABSTRACT

This paper deals with the kinematic and curvature analysis of the McPherson eight-bar suspension system that is developed by referring to a suitable planar car model, where both front wheels are supposed as rotating about their fixed revolute joints. In particular, the position analysis of the eight-bar mechanism, with an internal six-bar loop, is formulated to analyze the chassis roll motion by devoting particular attention to the roll center and its fixed and moving centrodes. Consequently, the curvature analysis is also carried out by using the Euler-Savary equation and the inflection circle, in order to determine the center of curvature of the mass center trajectory for the chassis roll motion. Finally, the proposed formulation has been implemented in a Matlab program and several examples allowed its validation in different mechanism configurations.

Keywords: Kinematics, linkages, McPherson suspension, roll-center, centrodes.

1. INTRODUCTION

The dynamic behavior of the vehicle is generally very difficult to model as it depends on many factors, including the design of the suspension system. The most popular suspension systems for commercial vehicles are certainly the double wishbone, the Multilink and the McPherson [1]. One of the main differences between these is that, compared to the first two, the McPherson suspension can absorb longitudinal and lateral forces and replaces the upper suspension link [2].

Over the years, designers have paid particular attention to the design of the suspension system as it can allow to obtain an increase in the performance [3], especially by carrying out an accurate kinematic synthesis of the system itself [4]. Since these systems are typically linkages, the first step is that of number synthesis [5], by which, it is possible to define the geometry of the mechanism that can allow to obtain not only maximum performance, but also the optimization of important parameters

such as controllability and stability [6]. Optimizing the parameters that govern the suspension system can also affect the steering of the vehicle, as described in [7]. Typically, the design of the most common suspension systems is carried out with reference to simplified planar kinematic models, although there are also scientific works that present a modeling in three-dimensional space, as reported in [8].

One of the most important parameters influencing vehicle dynamics is the roll center, which is also defined by the geometry of the suspension system [9]. The roll center and instantaneous centers of rotation are in general key concepts both in the analysis and synthesis of the suspension systems [10]. There are various definitions of the roll center of the suspension system [1], but anyway it is a fundamental tool for the dynamic analysis of the vehicle and for the characterization of its lateral motion [12-15]. The roll center is a fundamental concept for understanding the kinematics of the vehicle, but in order to have a broader view of the behavior of the system, the designer can make use of different tools, such as the centrodes, which capture the exact law of motion of the system itself. A first result of the centrodes of a wishbone suspension system is reported in [16]. The use of geometric loci, such as centrodes [17-18] or the Bresse's circles [19], in the field of the mechanisms design is fundamental to understand the kinematic properties of mechanical systems. The determination of the centrodes of a single-track model in plane motion is given in [20].

Another tool that can allow the designer to obtain useful information during the kinematic synthesis step is the curvature analysis, as reported in [21]. All these examples given so far refer to simplified planar models, but there are examples of application of the same theories extended to the three-dimensional case, both in the case of the analysis of suspension systems [22] and in the case of transmission systems [23].

This paper deals with the kinematic and curvature analysis of the McPherson eight-bar suspension system that is developed

by referring to a suitable planar car model. In particular, the position analysis is formulated to analyze the chassis roll motion and devoting particular attention to the roll center and its centrodcs. Consequently, the curvature analysis is also carried out, in order to determine the center of curvature of the mass center trajectory. Finally, the proposed formulation has been implemented in a Matlab program and several examples allowed its validation in different mechanism configurations.

2. PLANAR CAR MODEL

Referring to Fig. 1, a suitable planar car model is assumed as composed by eight rigid bodies, where the fixed frame 1 is represented by the transversal and straight road profile to which both front wheels 2 and 5 are supposed to be joined in A_0 and D_0 . Moreover, the quaternary link $BCEF$ (chassis 8) is connected to both wheels 2 and 5 by means of the corresponding McPherson independent suspension systems, which are composed by the oscillating arms 3 and 6 that are joined respectively, to C and F of 8 and to A and D of wheels 2 and 5. The left and right car shock absorbers take the kinematic form of the prismatic joints 2/4 and 5/7, which corresponding piston rods 4 and 7 are joined to 8 at points B and E , respectively. Assuming a rigid connection of wheels 2 and 5 to the ground 1 by means of the revolute joints A_0 and D_0 , the planar car model takes the form of an eight-bar mechanism with one d.o.f. that is composed by a quaternary link (chassis 8), two ternary links (wheels 2 and 5), four binary links (oscillating arms 3 and 6, along with the piston rods 4 and 7) and the fixed frame 1. Under a kinematic point of view, the chassis 8, as one of the five coupler links of the eight-bar mechanism, represents a quaternary coupler link and thus, it performs a general rigid body planar motion, by which the roll center is the instant center of rotation (IC) of 8.

The ICs are of great interest for analyzing the kinematic properties and performance of the eight-bar mechanism that is derived by the assumed car model.

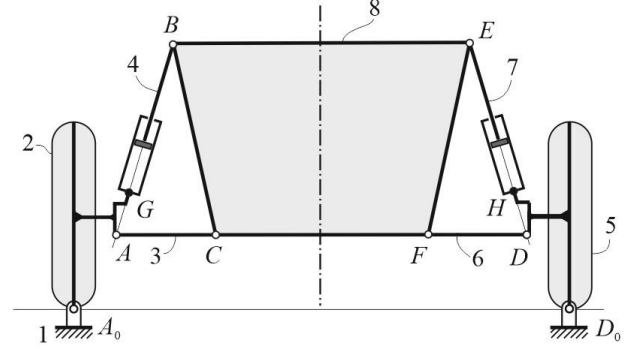


FIGURE 1: PLANAR CAR MODEL

Thus, referring to Fig. 2 and applying the Aronhold-Kennedy theorem [24], the total number n_{IC} of ICs can be determined by using the following equation

$$n_{IC} = \frac{n(n-1)}{2}$$

where n is the number of links. Thus, for $n = 8$, one has $n_{IC} = 28$.

In particular, P_2 and P_5 are the ICs of the wheels 2 and 5, P_{23} and P_{24} those between link 2/3 and 2/4, P_{38} and P_{48} those between links 3/8 and 4/8, P_{56} and P_{57} those between links 5/6 and 5/7, P_{68} and P_{78} those between 6/8 and 7/8.

Thus, applying the Aronhold-Kennedy theorem, P_{28} is the intersecting point between both lines $P_{24} - P_{48}$ and $P_{23} - P_{38}$, while P_{58} is obtained as intersecting point between the lines $P_{24} - P_{48}$ and $P_{23} - P_{38}$. Finally, the roll center P_8 is found as the intersecting point between the lines $P_2 - P_{28}$ and $P_5 - P_{58}$.

This point is fundamental to analyze the chassis motion in the transversal vertical plane of the car and to determine both fixed and moving centrodcs, as basis for the next curvature analysis.

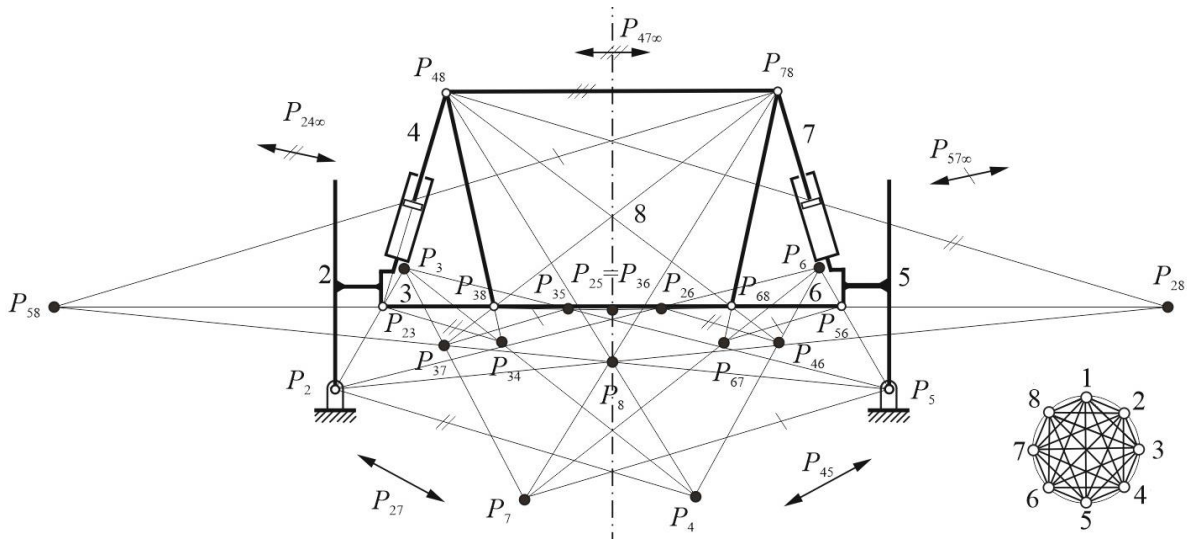


FIGURE 2: ALL ICs AND THE CORRESPONDING AK LINES

3. ROLL MOTION ANALYSIS

Referring to Fig.3, the position analysis is developed by the following three loop-closure equations

$$\mathbf{r}_{2A} + \mathbf{r}_2 + \mathbf{r}_4 + \mathbf{r}_{8a} + \mathbf{r}_7 = \mathbf{r}_1 + \mathbf{r}_{5D} + \mathbf{r}_5 \quad (1)$$

$$\mathbf{r}_{2A} + \mathbf{r}_3 + \mathbf{r}_{8b} + \mathbf{r}_6 = \mathbf{r}_1 + \mathbf{r}_{5D} \quad (2)$$

$$\mathbf{r}_2 + \mathbf{r}_4 = \mathbf{r}_3 + \mathbf{r}_{8c} \quad (3)$$

Consequently, the position vectors $\mathbf{r}_A, \mathbf{r}_B, \mathbf{r}_C, \mathbf{r}_D, \mathbf{r}_E, \mathbf{r}_F, \mathbf{r}_G, \mathbf{r}_H$, and $\mathbf{r}_{G8}$, of points A, B, C, D, E, F, G, H and G_8 are given by

$$\mathbf{r}_A = \mathbf{r}_{2A} \quad \mathbf{r}_C = \mathbf{r}_{2A} + \mathbf{r}_3 \quad \mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{8c} \quad \mathbf{r}_G = \mathbf{r}_A + \mathbf{r}_2 \quad (4)$$

$$\mathbf{r}_D = \mathbf{r}_{5D} \quad \mathbf{r}_H = \mathbf{r}_D + \mathbf{r}_5 \quad \mathbf{r}_E = \mathbf{r}_F + \mathbf{r}_{8c} \quad \mathbf{r}_F = \mathbf{r}_D + \mathbf{r}_6 \quad (5)$$

$$\mathbf{r}_{G8} = \mathbf{r}_B + \frac{1}{2}\mathbf{r}_{8a} + \frac{1}{3}\left(\frac{2r_{8a} + r_{8b}}{r_{8a} + r_{8b}}\right)\mathbf{h} \quad (6)$$

Thus, each vector $\mathbf{r}_i$ for $i = 1, \dots, 7$ can be expressed in matrix form with respect to the fixed frame OXY as

$$\mathbf{r}_i = r_i [\cos \theta_i \quad \sin \theta_i \quad 1]^T \quad (6)$$

where r_i and θ_i are the magnitude and the counterclockwise angle of vector $\mathbf{r}_i$ respectively, while T indicates the transpose, while r_{8b} and r_{8h} can be expressed as

$$\mathbf{r}_{8c} = r_{8c} [\cos(\theta_8 + \theta_0) \quad \sin(\theta_8 + \theta_0) \quad 1]^T \quad (7)$$

$$\mathbf{r}_{8a} = r_{8a} [\cos \theta_8 \quad \sin \theta_8 \quad 1]^T \quad (8)$$

$$\text{where } \theta_0 = \cos^{-1}\left(\frac{r_{8a} - r_{8b}}{2r_{8c}}\right).$$

Developing Eq. (1) – (3) one has

$$\begin{aligned} r_{2A} \cos \theta_2 - (r_2 + r_4) \cos(\beta_0 - \theta_2) + r_{8a} \cos \theta_8 - (r_5 - r_7) \cdot \\ \cos(\theta_5 + \beta_0) = r_1 + r_{5D} \cos \theta_5 \end{aligned} \quad (9)$$

$$\begin{aligned} r_{2A} \sin \theta_2 + (r_2 + r_4) \sin(\beta_0 - \theta_2) + r_{8a} \sin \theta_8 - (r_5 - r_7) \cdot \\ \sin(\theta_5 + \beta_0) = r_{5D} \sin \theta_5 \end{aligned}$$

$$\begin{aligned} r_{2A} \cos \theta_2 + r_3 \cos \theta_3 + r_{8b} \cos \theta_8 + r_6 \cos \theta_6 = r_1 + r_{5D} \cos \theta_5 \\ r_{2A} \sin \theta_2 + r_3 \sin \theta_3 + r_{8b} \sin \theta_8 + r_6 \sin \theta_6 = r_{5D} \sin \theta_5 \end{aligned} \quad (10)$$

$$\begin{aligned} (r_2 + r_4) \cos(\beta_0 - \theta_2) = r_3 \cos \theta_3 + r_{8c} \cos(\theta_8 + \theta_0) \\ -(r_2 + r_4) \sin(\beta_0 - \theta_2) = r_3 \sin \theta_3 + r_{8c} \sin(\theta_8 + \theta_0) \end{aligned} \quad (11)$$

Equations (9 – 11) represent a non-linear system, where r_i, θ_2, β_0 are given, while the oriented angles $\theta_3, \theta_5, \theta_6$ and θ_8 are the unknowns along with the link lengths r_4 and r_7 .

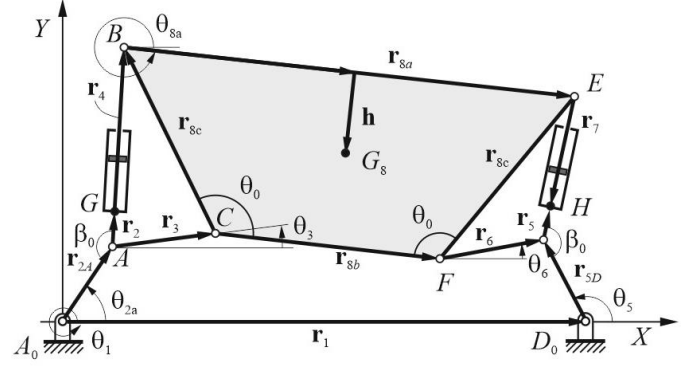


FIGURE 3: VECTOR LOOPS OF THE 8-BAR MECHANISM

This non-linear system has been solved by using the *fsolve* function of Matlab, which can give the solution without iterations. In particular, the *fsolve* function is able to find all the independent variables, such as those for which all the functions are zero or near zero. Thus, the choice of the initial configuration of the mechanism is important in order to obtain a meaningful result.

4. ROLL CENTER AND CENTRODES

In general, the relative planar motion between two rigid bodies can be reproduced by the pure rolling of two curves that take the role of centrodes, which are traced by the instantaneous center of rotation on their corresponding planes. If one of the two rigid bodies is fixed, a pair of fixed and moving centrodes can be obtained.

In particular, referring to Fig.4, the fixed centrode λ is the path traced by the instantaneous center of rotation P_8 of the body 8 with respect the fixed frame OXY , while the moving centrode l is the path traced by P_8 with respect the moving frame $O_8x_8y_8$ that is attached to link 8.

According to the Aronhold-Kennedy theorem, the fixed centrode λ of the body can be expressed by

$$\mathbf{r}_{8\lambda} = \mathbf{r}_{P8} = \begin{bmatrix} \frac{x_{P28}(x_{D0}y_{P58} - x_{P58}y_{D0})}{y_{P28}(x_{D0} - x_{P58}) - x_{P28}(y_{D0} - y_{P58})} \\ \frac{y_{P28}(x_{D0}y_{P58} - x_{P58}y_{D0})}{y_{P28}(x_{D0} + x_{P58}) - x_{P28}(y_{D0} - y_{P58})} \\ 1 \end{bmatrix} \quad (12)$$

where x_{P28} and y_{P28} can be found as the intersection of lines $P_{24} - P_{48}$ and $P_{23} - P_{38}$, one has

$$\mathbf{r}_{P28} = \begin{bmatrix} \frac{(x_A - x_C)(y_B^2 + x_B^2 - x_Ax_B) - x_A[y_A(y_B + y_C) - y_By_C] - x_Cy_A(y_A - 2y_B)}{(y_B - y_A)(y_A - y_C) + (x_B - x_A)(x_A - x_C)} \\ \frac{(y_A - y_C)(y_B^2 + x_B^2 - y_Ay_B) - y_A[x_A(x_B + x_C) - x_Bx_C] - y_Cx_A(x_A - 2x_B)}{(y_B - y_A)(y_A - y_C) + (x_B - x_A)(x_A - x_C)} \\ 1 \end{bmatrix} \quad (13)$$

while x_{P58} and y_{P58} can be found on the intersection of lines $P_{24} - P_{48}$ and $P_{23} - P_{38}$, as

$$\mathbf{r}_{P58} = \begin{bmatrix} \frac{(x_D - x_F)(y_E^2 + x_E^2 - x_D x_E) - x_D[y_D(y_E + y_F) - y_E y_F] - x_F y_D(y_D - 2y_E)}{(y_E - y_D)(y_D - y_F) + (x_E - x_D)(x_D - x_F)} \\ \frac{(y_D - y_F)(y_E^2 + x_E^2 - y_D y_E) - y_D[x_D(x_E + x_F) - x_E x_F] - y_F x_D(x_D - 2x_E)}{(y_E - y_D)(y_D - y_F) + (x_E - x_D)(x_D - x_F)} \\ 1 \end{bmatrix} \quad (14)$$

The moving centrodre l takes the form

$$\mathbf{r}_{8l} = \begin{bmatrix} \overline{P_8 C} \cos(\theta_{P8C} - \theta_8) & \overline{P_8 C} \sin(\theta_{P8C} - \theta_8) & 1 \end{bmatrix}^T \quad (15)$$

where

$$\overline{P_8 C} = \sqrt{(x_{P8} - x_C)^2 + (y_{P8} - y_C)^2} \quad (16)$$

$$\theta_{P8C} = \tan^{-1} \left(\frac{y_{P8} - y_C}{x_{P8} - x_C} \right) \quad (17)$$

In order to express the moving centrodre l of the quaternary link 8 with respect to OXY , the position vector $\mathbf{r}_{pi}^*$ of a generic point P with respect to the moving frame $\mathcal{Q}_i x_i y_i$ is given by

$$\mathbf{r}_{pi}^* = r_{pi}^* \begin{bmatrix} \cos \theta_i & \sin \theta_i & 1 \end{bmatrix}^T \quad (18)$$

and thus, the position vector $\mathbf{r}_{pi}$ of P in OXY is given by

$$\mathbf{r}_{pi} = \mathbf{T}_8 \mathbf{r}_{pi}^* \quad (19)$$

being $\mathbf{T}_8$ the following transformation matrix by the moving frame $O_i x_i y_i$ to OXY

$$\mathbf{T}_8 = \begin{bmatrix} \cos \theta_8 & -\sin \theta_8 & x_{O8} \\ \sin \theta_8 & \cos \theta_8 & y_{O8} \\ 0 & 0 & 1 \end{bmatrix} \quad (20)$$

where θ_i is the counterclockwise rotation angle of $O_i x_i y_i$ with respect to OXY , while x_{O_i} and y_{O_i} are the Cartesian coordinates of the origin O_i in the same fixed frame.

The coordinates $(x_{O\lambda}, y_{O\lambda})$ of a point of the osculating circle for the fixed centrodre λ can be found as the intersection of the following two straight lines

$$\begin{aligned} y - y_{M1} &= -\frac{1}{m_1} (x - x_{M1}) \\ y - y_{M2} &= -\frac{1}{m_2} (x - x_{M2}) \end{aligned} \quad (21)$$

Thus, one has

$$\mathbf{r}_{O\lambda} = \begin{bmatrix} \frac{(x_{M1} m_2 - x_{M2} m_1) + m_1 m_2 (y_{M1} - x_{M2})}{m_1 - m_2} \\ \frac{x_{M1} - x_{M2} + y_{M1} m_1 - x_{M2} m_2}{m_1 - m_2} \\ 1 \end{bmatrix} \quad (22)$$

where

$$m_1 = \frac{y_{P8}^{(i-1)} - y_{P8}^{(i)}}{x_{P8}^{(i-1)} - x_{P8}^{(i)}} \quad m_2 = \frac{x_{P8}^{(i+1)} - x_{P8}^{(i)}}{y_{P8}^{(i+1)} - y_{P8}^{(i)}} \quad (23)$$

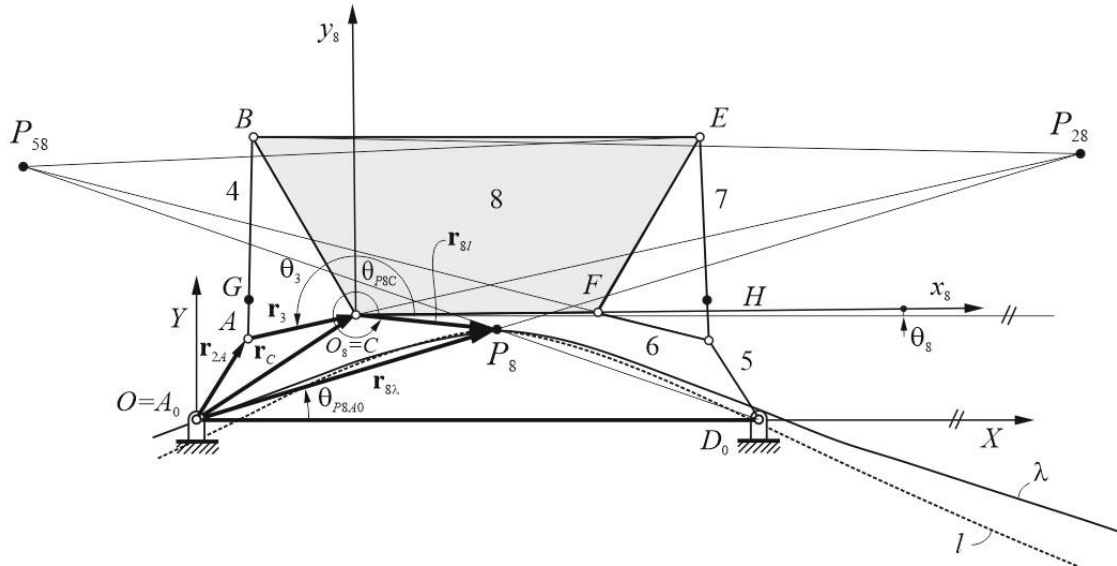


FIGURE 4: FIXED λ AND MOVING l CENTRODES

$$\begin{aligned} x_{M1} &= \frac{x_{P8}^{(i-1)} + x_{P8}^{(i)}}{2}, & y_{M1} &= \frac{y_{P8}^{(i-1)} + y_{P8}^{(i)}}{2} \\ x_{M2} &= \frac{x_{P8}^{(i+1)} + x_{P8}^{(i)}}{2}, & y_{M2} &= \frac{y_{P8}^{(i+1)} + y_{P8}^{(i)}}{2} \end{aligned} \quad (24)$$

The curvature radius r_λ of the fixed centrode λ can be determined as follows

$$r_\lambda = \sqrt{(x_{O\lambda} - x_{M1})^2 + (y_{O\lambda} - y_{M1})^2} \quad (25)$$

The same approach can be also applied to achieve the curvature radii of the moving centrode l and the trajectory of the mass center G_8 , respectively.

Thus, the diameter Δ of the inflection circle I can be determined by taking into what follows

$$-\frac{1}{\Delta} = \frac{1}{r_\lambda} - \frac{1}{r_l} \quad \Delta = \frac{r_\lambda r_l}{r_\lambda - r_l} \quad (26)$$

Consequently, the equation of I takes the form

$$(x - x_{OF})^2 + (y - y_{OF})^2 = \left(\frac{\Delta}{2}\right)^2 \quad (27)$$

where the Cartesian coordinates x_{OF} and y_{OF} of the center are expressed as function of the angle $\mathcal{G}$ by

$$x_{OF} = x_{P8} - \frac{\Delta}{2} \cos \mathcal{G}, \quad y_{OF} = x_{P8} - \frac{\Delta}{2} \sin \mathcal{G} \quad (28)$$

$$\mathcal{G} = \tan^{-1} \frac{y_{P8} - y_{O\lambda}}{x_{P8} - x_{O\lambda}} \quad (29)$$

Finally, the center of curvature Ω_{G8} of the path traced by the mass center G_8 of the chassis, can be obtained from the Euler-Savary equation, as follows

$$\Omega_{G8} G_8 = \frac{(P_8 G_8)^2}{G'_8 G_8} \quad (30)$$

where G'_8 is the corresponding point of G_8 that lies on the inflection circle.

5. GRAPHICAL AND NUMERICAL RESULTS

The proposed formulation has been implemented in Matlab and validated by means of significant examples.

In particular, Fig. 7 shows the eight-bar mechanism of the planar car model of Fig. 1 for $\theta_2 = 58.3^\circ$, $\omega_2 = 1$ r/s, $r_1 = 1500$ u, $r_2 = r_5 = 200$ u, $r_{2A} = r_{5D} = 250$ u, $r_3 = r_6 = 300$ u, $r_2 = r_5 = 100$ u,

$r_{8a} = 1200$ u, $r_{8b} = 650$ u, $r_{8c} = 550$ u, $\beta_0 = 150^\circ$, where u is the unit length, while Fig. 8 and 9 show other similar examples for $\theta_2 = 60^\circ$ and $\theta_2 = 56^\circ$, respectively.

In particular, Fig. 7 shows a mechanism configuration that is close to the symmetric one, where the fixed axis of symmetry coincides with that of the chassis 8. The main ICs that are strictly necessary to determine the position of the roll center P_8 are calculated and graphically represented, along with the corresponding alignments.

Moreover, both fixed and moving centrodes of the quaternary link (chassis 8) are also calculated and represented as a set of points, since traced by the roll center P_8 with respect to the fixed frame that is attached to the ground 1 and to the moving frame that is attached to the chassis 8.

Consequently, the pure rolling motion of the moving centrode l on the fixed centrode λ can be simulated by means of the proposed formulation, along with the eight-bar mechanism motion, which is evident by observing also the Figs. 8 and 9. In particular, Fig. 8 shows a rotated configuration, where the roll center P_8 is located on the right side, while Fig. 9 shows a mechanism configuration, which corresponding position of P_8 is on the left side of the fixed centrode.

In addition, the proposed curvature analysis that has been developed with the aid of the analytical geometry and thus, implemented in the same algorithm, allows the determination of the inflection circle for the coupler motion of the chassis 8 with respect to the ground 1 in any mechanism configuration. In fact, it is just the case to remind that this eight-bar mechanism shows five coupler links and for each of them, a pair of centrodes along with a proper inflection circle could be traced and analyzed.

The contribution of this paper is addressed to analyze the chassis rigid body motion, because this is the most relevant rigid body with respect to the others in terms of mass and inertia moment, for the dynamic analysis of the whole car motion. This is why, we are more interested in the chassis motion.

The inflection circle of the chassis 8 is not of easy determination, because the eight-bar mechanism of the car model shows a six-bar internal loop and consequently, the common application of the Euler-Savary equation, the Hartmann construction and the Bobillier theorem, are not sufficient.

This paper proposes a novel numerical method to determine the inflection circle of 8 by the centrodes, which are obtained by the roll center P_8 , as described above. Then particular attention has been devoted to the mass center G_8 , because this is the application point of the inertia forces.

Thus, the center and the radius of curvature of the mass center trajectory have been determined with the aid of the inflection circle and by using a direct numerical approach by means of the osculating circle at the mass center trajectory.

In particular, both centrodes have been obtained as a set of points, by which the corresponding osculating circles cannot be determined by using the differential geometry and this is why, a numerical approach has been applied by considering a circle passing through three consecutive points that are centered at the roll center P_8 in agreement with the procedure that is given by the Eqs. (21-25). According to the Euler-Savary equation (26),

the radius of the inflection circle I can be determined and consequently, the radius of curvature $r_{G8} = \Omega_{G8} G_8$ of the mass center G_8 is given by Eq. (30). This result is compared with that obtained by applying the same numerical approach, because G_8 traces a trajectory as set of points.

These numerical results have been compared between them at the symmetric configuration by giving the radii of curvature

$r_{G8} = 14.5108$ u and $r_{G8} = 14.3528$ u, which are almost identical between them, because of the optimal approximation that is given by the proposed method.

Finally, another fundamental aspect is that related to the stability analysis of the chassis motion, which is unstable because the current dynamic behavior of the G_8 mass center is that of an inverse pendulum.

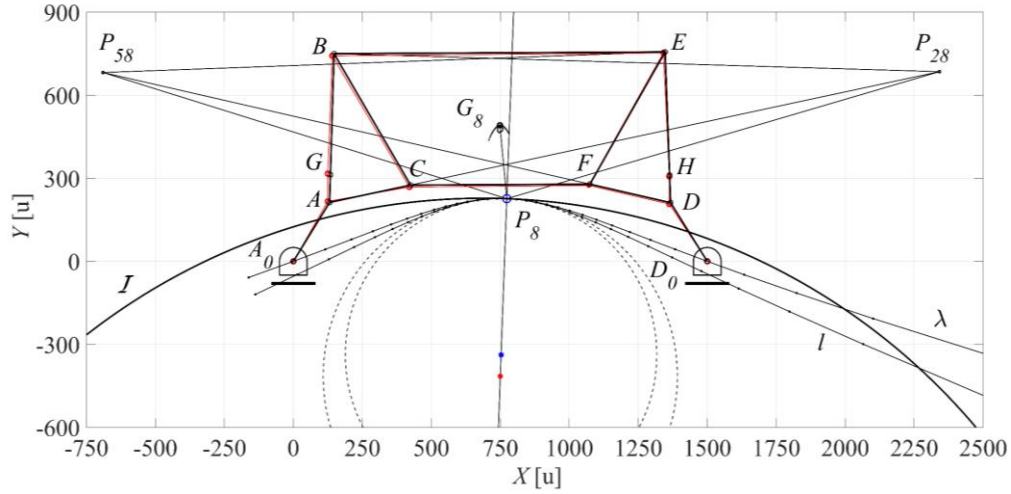


FIGURE 7: 8-BAR MECHANISM FOR $\theta_2 = 58.3^\circ$

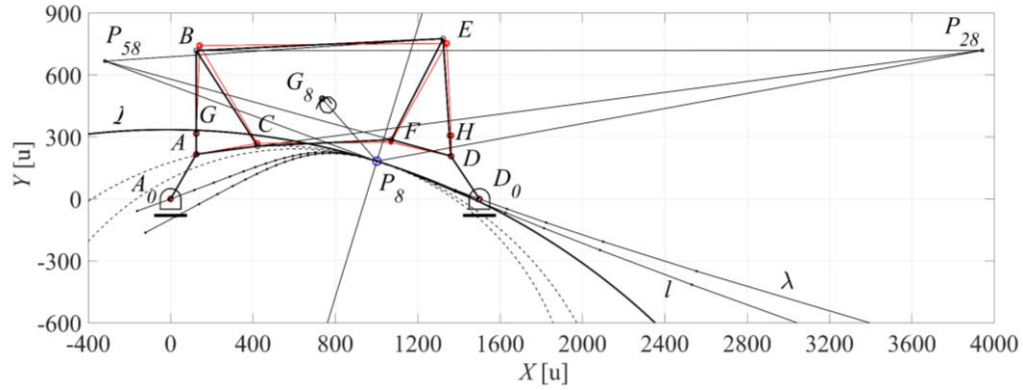


FIGURE 8: 8-BAR MECHANISM FOR $\theta_2 = 60^\circ$

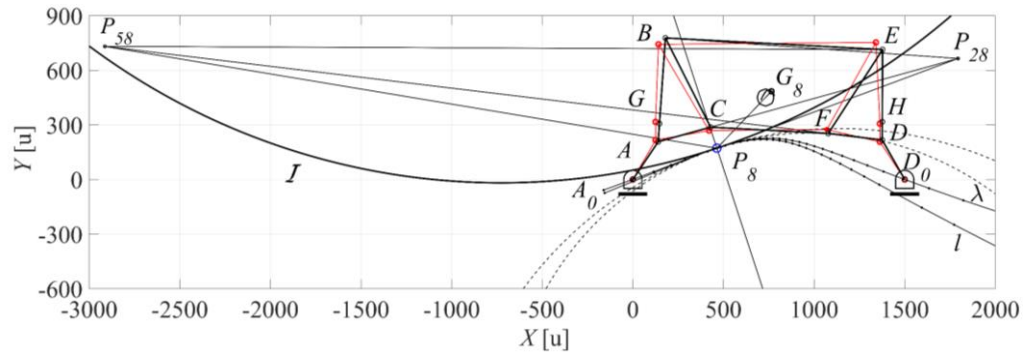


FIGURE 9: 8-BAR MECHANISM FOR $\theta_2 = 56^\circ$

6. CONCLUSIONS

The kinematic and the curvature analysis of the McPherson eight-bar suspension system is developed by referring to a suitable planar car model, where both front wheels are supposed as rotating about their fixed revolute joints. The position analysis of the eight-bar mechanism, with an internal six-bar loop, has been formulated with the aim to analyze the chassis roll motion by devoting particular attention to the roll center and to its fixed and moving centrodes, which are then used for the curvature analysis. In particular, this paper has been addressed to analyze the chassis rigid body motion, because this is the most important rigid body with respect to the others for the dynamic analysis of the whole car motion in terms of mass and inertia moment.

The proposed formulation has been implemented in a Matlab program and several examples allowed its validation in different mechanism configurations.

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